Teacher's Copy

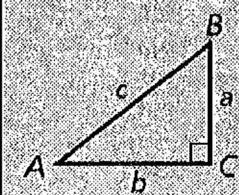
8.2 Trigonometric Ratios
Objectives:

Learning Goal: Students will find the sine, cosine, and tangent of an acute angle, and find side lengths in right triangles to solve real-world problems.

________________________ is a ratio of two sides of a right triangle.

SOH (Opposite, Hyp)
CAH (Adjacent, Hyp.)
TOA (Opp, Adj.)

Trigonometric Ratios

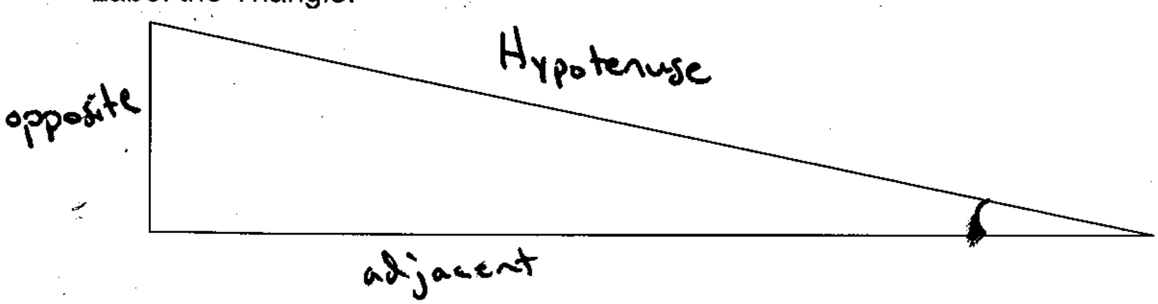
DEFINITION	SYMBOLS	DIAGRAM
The **sine** of an angle is the ratio of the length of the leg opposite the angle to the length of the hypotenuse.	$\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{a}{c}$ $\sin B = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{b}{c}$	
The **cosine** of an angle is the ratio of the length of the leg adjacent to the angle to the length of the hypotenuse.	$\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{b}{c}$ $\cos B = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{a}{c}$	
The **tangent** of an angle is the ratio of the length of the leg opposite the angle to the length of the leg adjacent to the angle.	$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{a}{b}$ $\tan B = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{b}{a}$	

Hypotenuse- Always opposite the right angle

Opposite side- Across from the angle involved in the calculation

Adjacent side- non hypotenuse side forming the angle used in the calculation

Label the Triangle:



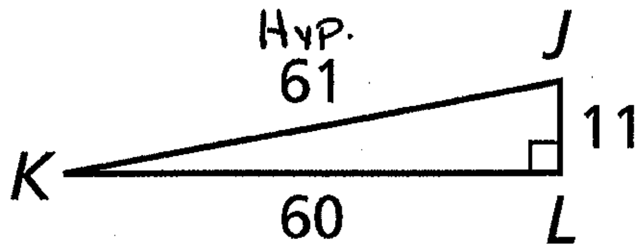
SOH CAH TOA

Example #1: Write the trigonometric ratio as a fraction and as a decimal rounded to the nearest hundredth.

sin J = $\frac{opp}{Hyp}$ = $\frac{60}{61}$ or .98

tan K = $\frac{opp}{adj.}$ = $\frac{11}{60}$ or .18

cos K = $\frac{Adj}{Hyp}$ = $\frac{60}{61}$ or .98

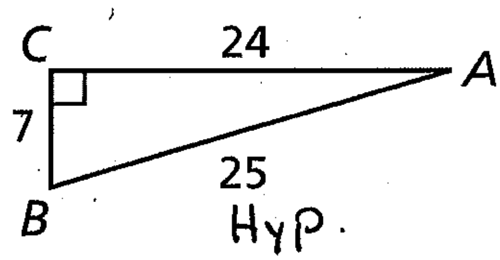


Example#2: Write the trigonometric ratio as a fraction and as a decimal rounded to the nearest hundredth.

cos A = $\frac{Adj}{Hyp}$ = $\frac{24}{25}$ or 0.96

tan B = $\frac{opp}{Adj}$ = $\frac{24}{7}$ or 3.43

sin B = $\frac{opp}{Hyp}$ = $\frac{24}{25}$ or 0.96



SOH
CAH
TOA

Example #3: Find the Given Trig Ratio.

sin 30 = $\frac{\cancel{s}}{2\cancel{s}}$

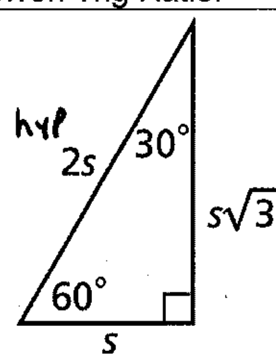
cos 60 $\frac{\cancel{s}}{2\cancel{s}}$

tan 30 $\frac{s}{s\sqrt{3}}$

tan 60 $\frac{s\sqrt{3}}{s}$

cos 30 $\frac{s\sqrt{3}}{2s}$

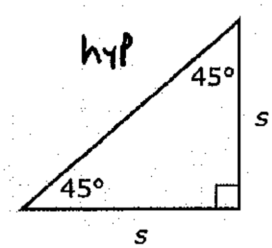
sin 60 $\frac{s\sqrt{3}}{2s}$



Sin 45 $\frac{s}{hyp}$

Cos 45 $\frac{s}{hyp}$

Tan 45 $\frac{s}{s}$ = 1



Example #4: Use your calculator to find the trigonometric ratio. Round to the nearest hundredth.

cos 19° = 0.95

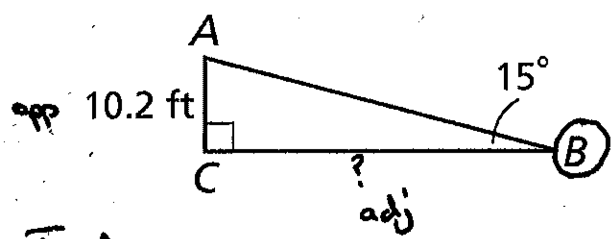
tan 11° = 0.19

sin 88° = 1.0

SoH
CAH
ToA

Example #5:

Find the length of BC.



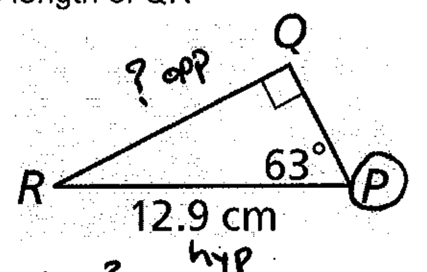
ToA

$\tan(15°) = \frac{10.2}{?}$ $\tan(15) = \frac{10.2}{x}$

- if x is on bottom
- switch the tan(15) w/ x to solve

$x = \frac{10.2}{\tan(15)}$ $x \approx 38.07$

Find the length of QR



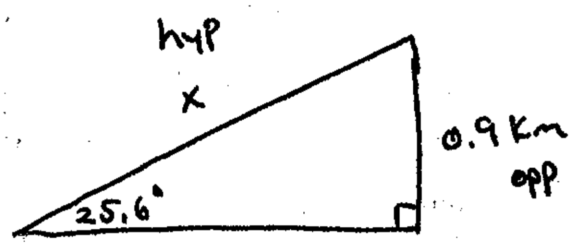
$\sin(63°) = \frac{?}{12.9}$ $\sin(63) = \frac{x}{12.9}$

- If x is on top
- multiply sin(63) and bottom number

$\sin(63) \cdot 12.9 = x$ $x = 11.49$

Example #6:

The Pilatusbahn in Switzerland is the world's steepest cog railway. Its steepest section makes an angle of about 25.6° with the horizontal and rises about 0.9 km. To the nearest hundredth of a kilometer, how long is this section of the railway track?



$\sin(25.6) = \frac{0.9}{x}$

$\frac{0.9}{\sin(25.6)} = 2.08$ km

HW 55 HW: Page 529: 22-48Even, 49-50

8.3 Solving Right Triangles

Learning Goal: Students will use trigonometric ratios to find angle measures in right triangles and solve real-world problems.

Inverse trig functions are used when we are trying to find the measurement of an angle.

Inverse Trigonometric Functions
If $\sin A = x$, then $\sin^{-1} x = m\angle A$.
If $\cos A = x$, then $\cos^{-1} x = m\angle A$.
If $\tan A = x$, then $\tan^{-1} x = m\angle A$.

Use your calculator to find each angle measure to the nearest degree.

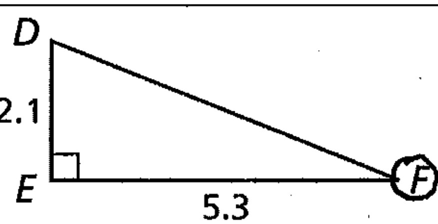
1a. $\cos^{-1}(0.87) \approx 30^\circ$ 1b. $\sin^{-1}(0.85) \approx 58^\circ$ 1c. $\tan^{-1}(0.71) \approx 35^\circ$

Find the unknown measures. Round the side lengths to the nearest hundredth and angle measures to the nearest degree.

SOH
CAH
TOA

Example 2

Find $\angle F$

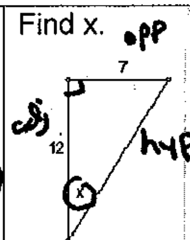


$\tan^{-1}(F) = \frac{2.1}{5.3}$

$m\angle F = \tan^{-1}\left(\frac{2.1}{5.3}\right)$

$m\angle F \approx 22^\circ$

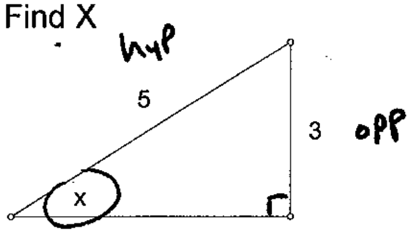
Find x.



$\tan^{-1}\left(\frac{7}{12}\right)$

$x \approx 30^\circ$

Find X



$\sin^{-1}\left(\frac{3}{5}\right)$

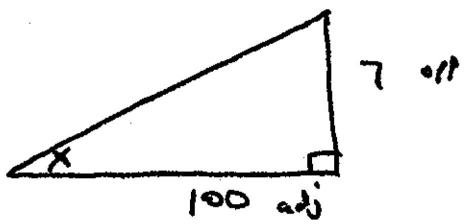
$m\angle x \approx 37^\circ$

The steepness of a road is often expressed as a percent grade. For example, a road with a grade of 6% actually raises 6 ft for every 100 ft it travels horizontally. $TanA = \frac{rise}{run} = roadgrade$

A highway sign warns that a section of road ahead has a 7% grade. To the nearest degree, what angle does the road make with a horizontal line?

TOA

$7\% = \frac{7 \text{ ft. rise}}{100 \text{ ft. run}}$

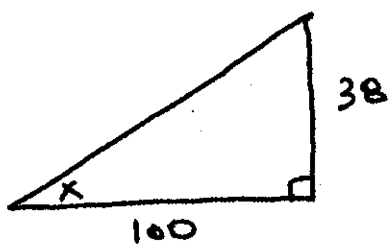


$\tan^{-1}\left(\frac{7}{100}\right)$

$m\angle x \approx 40^\circ$

Baldwin St. in Dunedin, New Zealand, is the steepest street in the world. It has a grade of 38%. To the nearest degree, what angle does Baldwin St. make with a horizontal line?

$38\% = \frac{38 \text{ ft rise}}{100 \text{ ft run}}$



$\tan^{-1}\left(\frac{38}{100}\right) \approx 21^\circ$

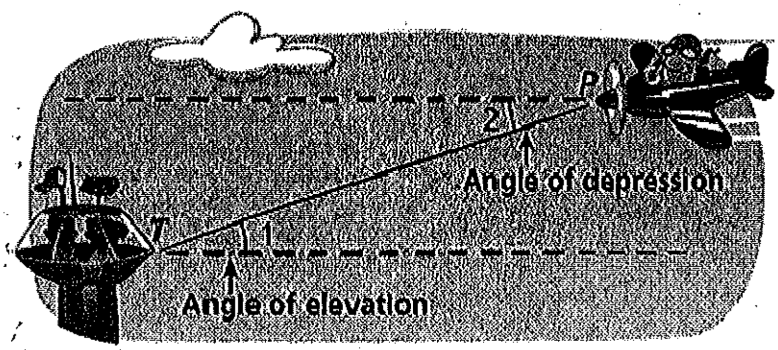
HW 56

HW: Page 538 21-36, 38-44

8.4 Angle of Elevation and Depression Objectives

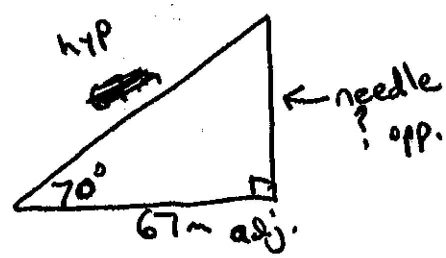
Learning Goal: Students will solve problems involving angles of elevation and depression.

An $\angle$ of elevation is the angle formed by a horizontal line and a line of sight to a point *above* the line. In the diagram, $\angle 1$ is the angle of elevation from the tower T to the plane P.
An $\angle$ of Depression is the angle formed by a horizontal line and a line of sight to a point *below* the line. $\angle 2$ is the angle of depression from the plane to the tower.



Ex. 1
The Seattle Space Needle casts a 67-meter shadow. If the angle of elevation from the tip of the shadow to the top of the Space Needle is 70°, how tall is the Space Needle? Round to the nearest meter.

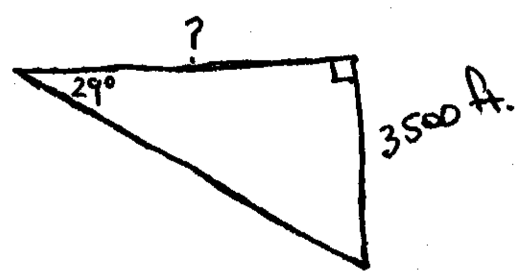
SOH
CAH
TOA



$\tan(70) = \frac{?}{67}$

$\tan(70) \cdot 67 = 184$ Km

Ex. 2
Suppose the plane is at an altitude of 3500 ft and the angle of elevation from the airport to the plane is 29°. What is the horizontal distance between the plane and the airport? Round to the nearest foot.



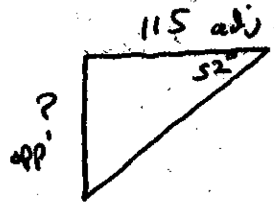
$\tan(29) = \frac{3500}{?}$

$\frac{3500}{\tan(29)} = 6314$ ft.

Ex. 3

TOA

An ice climber stands at the edge of a crevasse that is 115 ft wide. The angle of depression from the edge where she stands to the bottom of the opposite side is 52°. How deep is the crevasse at this point? Round to the nearest foot.

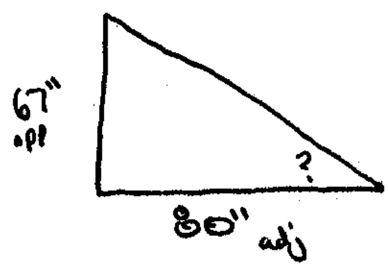


$\tan(52) = \frac{?}{115}$

$\tan(52) \cdot 115 = 147$ ft.

Ex. 4

Ms. Blay is 67" tall. At 3:00 in the afternoon, she notices her shadow is 80". Curious..., she wants to know the angle of elevation of the sun. Can you help her?



$\tan^{-1}\left(\frac{67}{80}\right) = 40°$

HW 57

HW: Page 547: 10-22

8.5 The Law of Sines and The Law of Cosines
Objectives

Learning Goal: Students will use the Law of Sines and the Law of Cosines to solve any triangle.

In 8.5 we learn to solve <u>ANY</u> Triangle (all triangles are NOT right triangles)
Some triangles have OBTUSE angles.

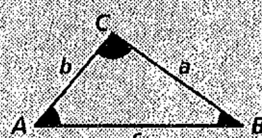
Use a calculator to find each trigonometric ratio. Round to the nearest hundredth.

1a. tan 175° ≈ −0.09 **1b. cos 92°** ≈ −0.03 **1c. sin 160°** ≈ 0.34

Theorem 8-5-1 The Law of Sines

For any $\triangle ABC$ with side lengths a, b, and c,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

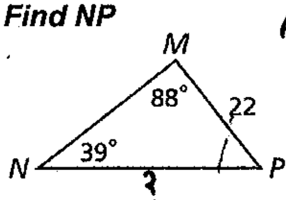
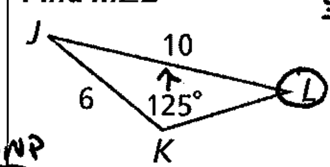
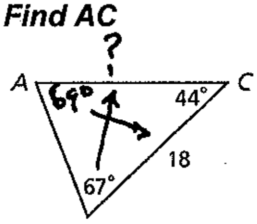


You can use the Law of Sines to solve a triangle if you are given:

- two angle measures and any side length (ASA or AAS)

  or
- two side lengths and a non-included angle measure (SSA).

Example 2: Find all the measures. Round lengths to the nearest tenth and angle measures to the nearest degree.

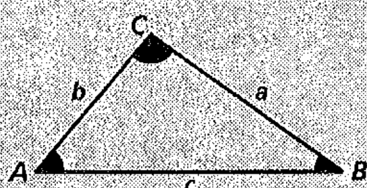
Find NP  AAS $\frac{\sin(39)}{22} = \frac{\sin(88)}{NP}$ $\frac{\sin(88)\cdot 22}{\sin(39)} = \frac{\sin(39)\cdot NP}{\sin(39)}$ $NP = 34.9$	***Find m∠L***  $\frac{\sin(125)}{10} = \frac{\sin(L)}{6}$ $\frac{6\cdot\sin(125)}{10} = \sin(L)$ $m\angle L = \sin^{-1}\left(\frac{6\cdot\sin 125}{10}\right)$ $m\angle L = 29°$
Find AC  $\frac{\sin(67)}{AC} = \frac{\sin(69)}{18}$ $\frac{\sin(67)\cdot 18}{\sin(69)} = 17.7$	− sin of the ∠ over its opp. side (equal to) − sin of the other ∠ over its opp. side
HW58 **Homework: P555 1-9, 10-12**	

* when finding the ∠ measure use $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$

Theorem 8-5-2 The Law of Cosines

For any $\triangle ABC$ with side lengths a, b, and c:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

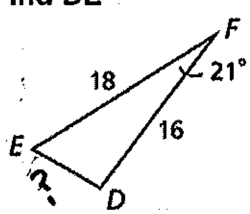


You can use the Law of Cosines to solve a triangle if you are given

- two side lengths and the included angle measure (SAS) or
- three side lengths (SSS).

Find the measure. Round lengths to the nearest tenth and angle measures to the nearest degree.

Find DE

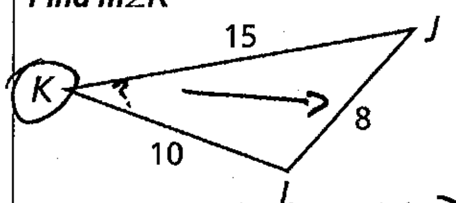


$ED^2 = 18^2 + 16^2 - 2(18)(16)\cos(21)$

$\sqrt{ED^2} = \sqrt{42.25}$

$ED = 6.5$

Find m∠K



$8^2 = 10^2 + 15^2 - 2(10)(15)\cos K$

$64 = 325 - 300 \cdot \cos K$

$-325 \quad -325$

$\frac{-261}{-300} = \frac{-300}{-300} \cdot \cos K$

$m\angle K = \cos^{-1}\left(\frac{261}{300}\right)$

$m\angle K = 30°$

$\left(\underset{\angle}{\text{side across}}\right)^2 = (\text{other side})^2 + (\text{other side})^2 - 2(\ )(\ )\cos(\angle)$

other sides again

HW 59 Homework: Page 555, 17-25, 13-15