Teacher's Copy

9.1 Triangles and Special Quadrilaterals

Learning Objective: Students will develop and apply the formulas for the area and perimeter of triangles and special quadrilaterals.

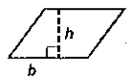
Postulate 9-1-1 **Area Addition Postulate**

The area of a region is equal to the sum of the areas of its nonoverlapping parts.

A triangle is cut off one side and translated to the other side.

Area **Parallelogram**

The area of a parallelogram with base *b* and height *h* is $A = bh$.

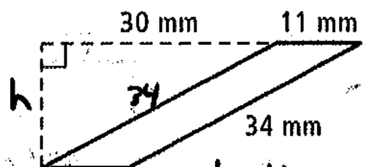


Quick Write:

Describe how to find the area of a rectangle: Length times width

Describe how to find the perimeter of a rectangle: Add all the side lengths up.

Find the area: $A = bh$



$a^2 + b^2 = c^2$

$h^2 + 30^2 = 34^2$

$h^2 + 900 = 1156$

$\sqrt{h^2} = \sqrt{256}$

$h = 16$

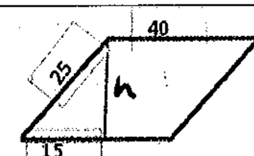
$b = 11$

$h = 16$

$A = 11(16)$

$A = 176\text{mm}^2$

Find the area:



$15^2 + h^2 = 25^2$

$225 + h^2 = 625$

$\sqrt{h^2} = \sqrt{400}$

$h = 20$

$h = 20$

$b = 40$

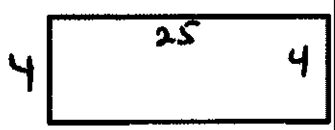
$A = 20(40)$

$A = 800$

Find the perimeter of the rectangle with an area of 100 cm².

Height = 4

Perimeter = 58



25 cm

$b = 25$

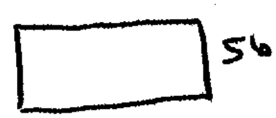
$\frac{100}{25} = \frac{25(h)}{25}$

$h = 4$

$P = 2 \cdot 4 + 2 \cdot 25$

$P = 58$

Find the base of the parallelogram in which $h = 56$ yd and $A = 28\text{ yd}^2$.



$\frac{28}{56} = \frac{b(56)}{56}$

$b = 0.5$

Find the perimeter of a square with an Area of 225 m².

Side length = 15m

Perimeter = $15 \cdot 4 = 60$ m

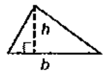
$A = s^2$

$\sqrt{225} = \sqrt{s^2}$

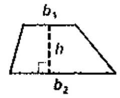
$s = 15$

Area Triangles and Trapezoids

The area of a triangle with base b and height h is $A = \frac{1}{2}bh$.



The area of a trapezoid with bases b_1 and b_2 and height h is $A = \frac{1}{2}(b_1 + b_2)h$, or $A = \frac{(b_1 + b_2)h}{2}$.

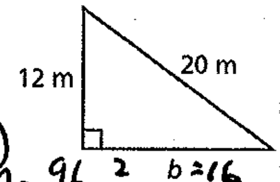


Find the area of a trapezoid in which b_1 = 8 in., b_2 = 5 in., and h = 6.2 in.

$A = \frac{(8+5)6.2}{2}$

$A = 10.3 \text{ in}^2$

Find the area of the triangle



$12^2 + b^2 = 20^2$

$144 + b^2 = 400$

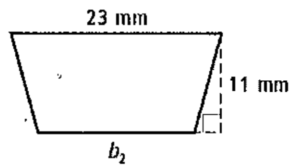
$\sqrt{b^2} = \sqrt{256}$

$b = 16$

$A = \frac{1}{2}(16)(12)$

$A = 96 m^2$

Find b_2 of the trapezoid, in which A = 231 mm².



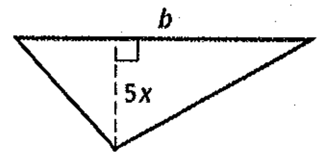
$2 \cdot (231) = \left(\frac{(23+b_2)11}{\cancel{2}}\right) \cdot 2$

$\frac{462}{11} = \frac{(23+b_2)\cancel{11}}{\cancel{11}}$

$42 = 23 + b_2$

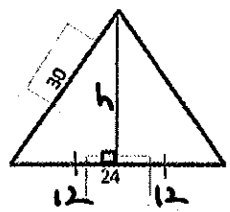
$b_2 = 19 \text{ mm}$

Find the base of the triangle, in which $A = (15x^2)$ cm².



$15x^2 = \frac{1}{2}(b)(5x)$

Find the area of the triangle



$12^2 + h^2 = 30^2$

$144 + h^2 = 900$

$\sqrt{h^2} = \sqrt{756}$

$h = 27.49$

$A = \frac{1}{2}(24)(27.49) = 329.88$

Hershey is trying to decide on the best shape for a new candy bar. The prototype is a square bar with sides measuring 4 cm. The research team has decided to try a rectangular shaped bar with an equivalent area to the original prototype. If the width of the rectangle is 2 cm, what will be the length of the new candy bar?

4 4 8 2

A: 16 A: 16

$l = 8$cm

You want to put wallpaper on 1 wall of your bedroom that is 14' x 9'. If each roll costs $ 6.55 and will cover 56 ft², how much will it cost to do the wall? How much wallpaper will you have left over?

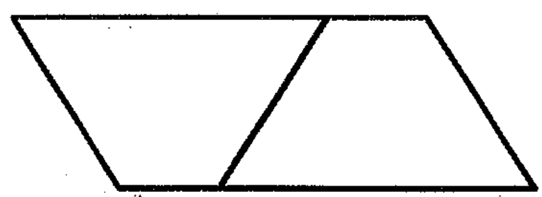
$56 \times 3 = 168$

3 rolls \$19.65

$168 - 126 = 42 \text{ ft}^2$

14 9

$A = 9 \cdot 14 = 126 \text{ ft}^2$



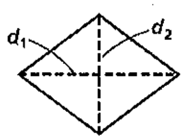
Quickwrite: How is the area of the trapezoid related to the area of the total figure created?

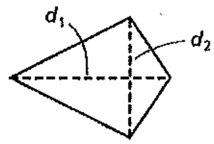
HW: WorkSheet

The diagonals of a rhombus or kite are $\perp$, and the diagonals of a rhombus bisect each other.

Area **Rhombuses and Kites**

The area of a rhombus or kite with diagonals d_1 and d_2 is $A = \frac{1}{2}d_1d_2$.





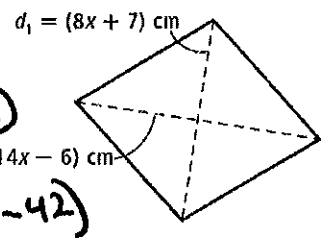
Find d_2 of a kite in which d_1 = 14 in. and A = 238 in^2.

$238 = \frac{1}{2}(14)\, d_2$

$\frac{238}{7} = \frac{7\, d_2}{7}$

$d_2 = 34$ in.

Find the area of the Rhombus:



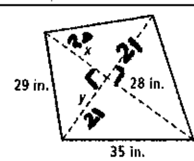
$A = \frac{1}{2}(8x+7)(14x-6)$

$A = \frac{1}{2}(112x^2 - 48x + 98x - 42)$

$A = \frac{1}{2}(112x^2 + 50x - 42)$

$A = (56x^2 + 25x - 21)\text{ cm}^2$

Find the area of the Kite:



$28^2 + b^2 = 35^2$ $\quad d_1 = 42$

$784 + b^2 = 1225$ $\quad d_2 = 48$

$\sqrt{b^2} = \sqrt{441}$

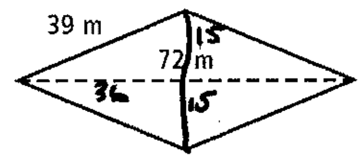
$b = 21$

$A = \frac{1}{2}(42)(48)$

$A = 1008\text{ in}^2$

$21^2 + b^2 = 29^2$

$441 + b^2 = 841$ $\quad \sqrt{b^2} = \sqrt{400}$ $\quad b = 20$



$36^2 + b^2 = 39^2$

$1296 + b^2 = 1521$

$\sqrt{b^2} = \sqrt{225}$

$b = 15$

$A = \frac{1}{2}(72)(30)$

$A = 1080\text{ m}^2$

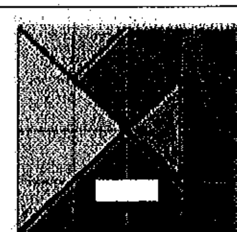
Find d_2 of a rhombus in which $d_1 = 3x$ m and $A = 12xy$ m^2.

$\frac{12xy}{3x} = \frac{\frac{1}{2}(3x)(d_2)}{3x}$

$\frac{4y}{\frac{1}{2}} = \frac{\frac{1}{2}(d_2)}{\frac{1}{2}}$

$8y = d_2$

In the tangram, find the perimeter and area of the large green triangle. Each grid square has a side length of 1 cm.



$h = 2$

$b = 4$

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(4)(2)$

$A = 4\text{ cm}^2$

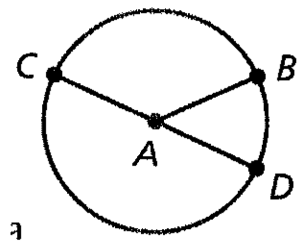
$P = (4 + 4\sqrt{2})\text{ cm}$

HW: WorkSheet

9.2 Circles and Regular Polygons

Learning Goal: Students will develop and apply the formulas for the area and circumference of a circle and a regular polygon.

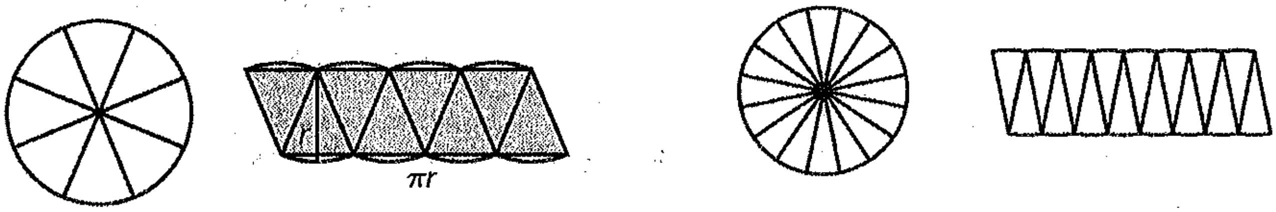
A <u>Circle</u> is the locus of points in a plane that are a fixed distance from a point called the <u>Center of the circle</u>. A circle is named by the symbol ⊙ and its center. **⊙*A*** has radius ***r*** = ***AB*** and diameter ***d*** = ***CD***.



The irrational number π is defined as the ratio of the circumference *C* to the diameter *d*, or $\pi = C/d$.

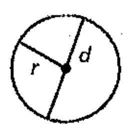
Solving for *C* gives the formula $C = \pi d$. Also $d = 2r$, so $C = 2\pi r$.

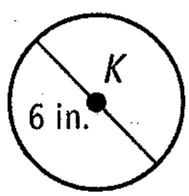
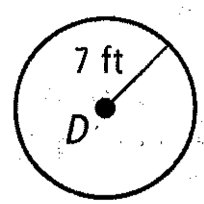
You can use the circumference of a circle to find its area. Divide the circle and rearrange the pieces to make a shape that resembles a parallelogram.



Circumference and Area **Circle**

A circle with diameter *d* and radius *r* has circumference $C = \pi d$ or $C = 2\pi r$ and area $A = \pi r^2$.



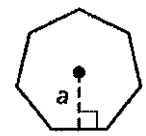
Find the area of ⊙*K* in terms of π.  $A = \pi r^2$ $A = 3^2\pi$ $A = 9\pi \text{ in}^2$	Find the area of ⊙*D* in terms of π  $A = \pi r^2$ $A = 49\pi \text{ ft}^2$
Find the circumference of ⊙*M* if the area is $25x^2\pi \text{ ft}^2$ $A = \pi r^2$ $25x^2\pi = \pi r^2$ $r = 5x$ $C = 2(5x)\pi$ $C = 10x\pi \text{ ft.}$	Find the radius of ⊙*J* if the circumference is $(65x + 14)\pi$ m. $C = 2\pi r$ $\frac{(65x+14)\pi}{2} = \frac{2\pi r}{2}$ $(32.5x + 7) = r$
Find the area of ⊙*A* in terms of π in which $C = (4x - 6)\pi$ **m.** $C = 2\pi r$ $\frac{(4x-6)\pi}{2} = \frac{2\pi r}{2}$ $(2x - 3) = r$ $A = (2x-3)^2\pi$ $A = (2x-3)(2x-3)$ $A = 4x^2 - 6x - 6x + 9$ $A = (4x^2 - 12x + 9)\pi \text{ m}^2$	**A pizza-making kit contains three circular baking stones with diameters 24 cm, 36 cm, and 48 cm. Find the area of each stone. Round to the nearest tenth.** $A = 12^2\pi = 452.4 \text{ cm}^2$ $A = 18^2\pi = 1017.8 \text{ cm}^2$ $A = 24^2\pi = 1809.6 \text{ cm}^2$

The **center of a regular polygon** is equidistant from the vertices. The apothem is the distance from the center to a side. A **central angle of a regular polygon** has its vertex at the center, and its sides pass through consecutive vertices. Each central angle measure of a regular *n*-gon is $360/n$.

Regular pentagon *DEFGH* has a center *C*, apothem *BC*, and central angle $\angle DCE$.

Area **Regular Polygon**

The area of a regular polygon with apothem *a* and perimeter *P* is $A = \frac{1}{2}aP$.



Find the area of regular heptagon with side length 2 ft to the nearest tenth.

$360/5 = 72 \div 2 = 36°$

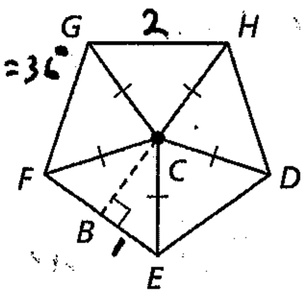
$\tan 36 = \frac{1}{a}$

$a = \frac{1}{\tan 36} = 1.38$

$A = \frac{1}{2}\left(\frac{1}{\tan(36)}\right) 10$

$A = 6.9\ cm^2$

$P = 5(2) = 10$



Find the area of a regular dodecagon with side length 5 cm to the nearest tenth.

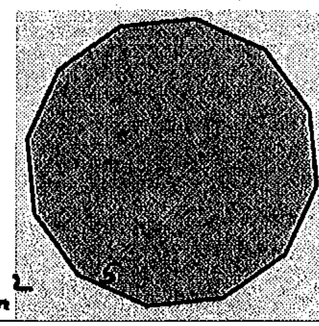
$P = 12 \cdot 5 = 60$

$\frac{360}{12} = 30 \div 2 = 15°$

$\frac{2.5}{\tan 15} = a$

$A = \frac{1}{2}\left(\frac{2.5}{\tan 15}\right) 60$

$A = 279.9\ cm^2$



Find the area of a regular pentagon with side length 6 cm to the nearest tenth.

$\frac{360}{5} = 72 \div 2 = 36°$

$a = \frac{3}{\tan 36} = 4.13$

$A = \frac{1}{2}\left(\frac{3}{\tan 36}\right) 30$

$A = 61.9\ cm^2$

$P = 5 \cdot 6 = 30$

Find the area of a regular hexagon with side length 8 cm to the nearest tenth.

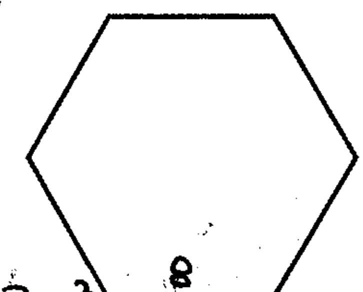
$P = 6 \cdot 8 = 48\ cm$

$\frac{360}{6} = 60 \div 2 = 30°$

$\frac{4}{\tan 30} = a$

$A = \frac{1}{2}\left(\frac{4}{\tan 30}\right) 48$

$A = 166.3\ cm^2$



Find the area of a regular octagon with a side length of 4 cm.

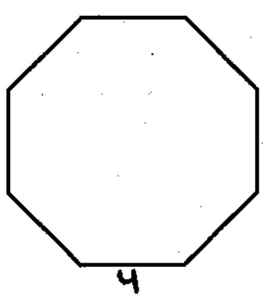
$P = 4 \cdot 8 = 32$

$\frac{360}{8} = 45 \div 2 = 22.5°$

$a = \frac{2}{\tan 22.5}$

$A = \frac{1}{2}\left(\frac{2}{\tan 22.5}\right) 32$

$A = 77.3\ cm^2$



HW: Worksheet Day 1

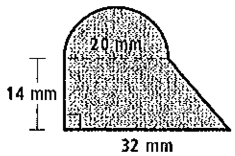
HW: Worksheet Day 2

9.3 Composite Figures

Learning Goal: Students will use the Area Addition Postulate to find the areas of composite figures and estimate the areas of irregular shapes.

A Composite figure is made up of simple shapes, such as triangles, rectangles, trapezoids, and circles. To find the area of a composite figure, find the areas of the simple shapes and then use the Area Addition Postulate.

Find the shaded area. Round to the nearest tenth, if necessary.



Circle $= \frac{1}{2}(\pi r^2)$

$\frac{1}{2}(10^2)\pi$

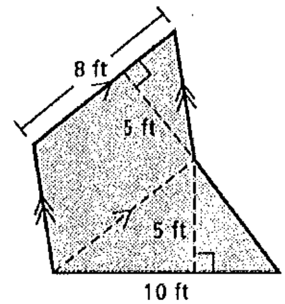
Circle $= \cancel{200}\ 50\pi$

Trapezoid $= \frac{1}{2}(20+32)\,14$

Trap $= 364$

Composite $= 521.1\ mm^2$

Find the shaded area. Round to the nearest tenth, if necessary.



$\triangle: \frac{1}{2}bh$

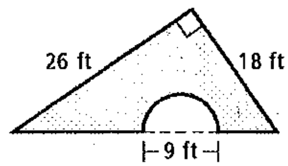
$\frac{1}{2}(10)(5) = 25\ ft^2$

▱ $A = bh$

$8 \cdot 5 = 40\ ft^2$

$25 + 40 = 65\ ft^2$

Find the shaded area. Round to the nearest tenth, if necessary.



Circle $= \frac{1}{2}(4.5^2)\pi$

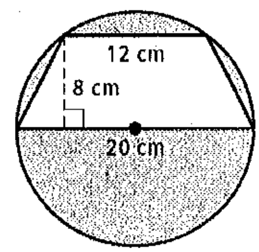
$= \frac{1}{2}(4.5^2)\pi$

$\triangle = \frac{1}{2}(26)(18)$

$\triangle = 234$

$234 - 31.8 = 202.2\ ft^2$

Find the shaded area. Round to the nearest tenth, if necessary.



⊙ $A = \pi r^2$

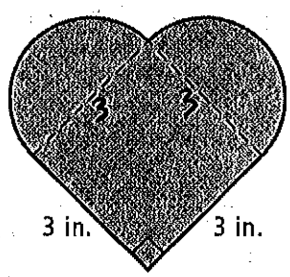
$= 10^2\pi = 314.16$

⏢ $= \frac{(b_1+b_2)}{2}h$

$\frac{(12+20)}{2}8 = 128$

$314.16 - 128 = 186.16\ cm^2$

A company receives an order for 65 pieces of fabric in the given shape. Each piece is to be dyed red. To dye 6 in² of fabric, 2 oz of dye is needed. How much dye is needed for the entire order?



⊙ $A = \pi r^2$

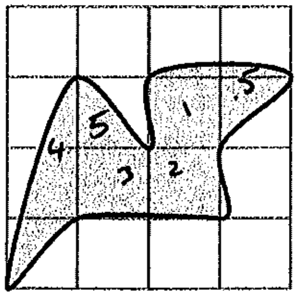
$A = (1.5)^2\pi = 7.07\ in^2$

□ $A = s^2$

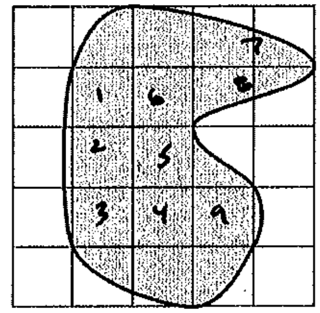
$3^2 = 9\ in^2$

$7.07 + 9 = 16.07\ in^2 \times 65 = 1044.55$

$174.09 \times 2 = 348\ oz.$



$\approx 5.5\ ft^2$



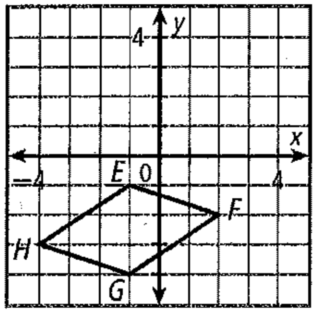
$\approx 12\ ft^2$

HW: Worksheet

9.4 Area and Perimeter in the Coordinate Plane

Learning Goal: Students will find the perimeters and areas of figures in a coordinate plane.

Draw and classify the polygon with vertices $E(-1, -1)$, $F(2, -2)$, $G(-1, -4)$, and $H(-4, -3)$. Find the perimeter and area of the polygon.



$A = bh$

$b = GF$

$h = HG$

$b = GF = \sqrt{(2+1)^2 + (-2+4)^2} = \sqrt{13}$

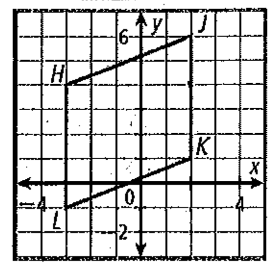
$h = HG = \sqrt{(-4-1)^2 + (-4-3)^2} = \sqrt{10}$

Perimeter $P = 2b + 2h =$

$2\sqrt{10} + 2\sqrt{13}$

Area $= bh = 9$ units^2

Draw and classify the polygon with vertices $H(-3, 4)$, $J(2, 6)$, $K(2, 1)$, and $L(-3, -1)$. Find the perimeter and area of the polygon.



$b = LK$

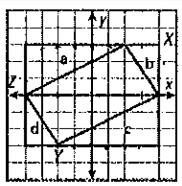
$h = JK$

$LK = \sqrt{(-3-2)^2 + (-1-1)^2} = \sqrt{29}$

$JK = \sqrt{(2-2)^2 + (6-1)^2} = \sqrt{25} = 5$

$P =$

Find the area of the polygon with vertices $A(-4, 1)$, $B(2, 4)$, $C(4, 1)$, and $D(-2, -2)$.



$A = bh = 6(8) = 48$ units^2

$a = A = \frac{1}{2}bh = \frac{1}{2}(6)(3) = 9$

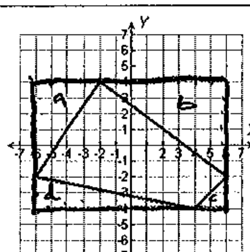
$b: \frac{1}{2}(2)(3) = 3$

$c: \frac{1}{2}(3)(6) = 9$

$d: \frac{1}{2}(2)(3) = 3$

$48 - 3 - 3 - 9 - 9 = 24$ units^2

Find the area of the polygon with vertices $K(-2, 4)$, $L(6, -2)$, $M(4, -4)$, and $N(-6, -2)$.



$A = bh = (8)(12) = 96$

$a: \frac{1}{2}(6)(4) = 12$

$b: \frac{1}{2}(8)(6) = 24$

$c: \frac{1}{2}(2)(2) = 2$

$d: \frac{1}{2}(2)(9) = 8$

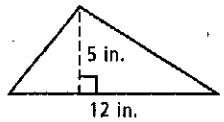
$96 - 12 - 24 - 2 - 8 = 48$ units^2

HW: Page 620, 10-22

9.5 Dimensional Change

Learning Goal: Students will describe the effect on perimeter and area when one or more dimensions of a figure are changed and apply the relationship between perimeter and area in problem solving.

Describe the effect of each change on the area of the given figure:



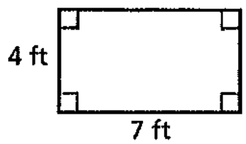
The height of the Δ is multiplied by 6.

The area is multiplied by 6

The diagonal *SU* of the kite with vertices *R*(2, 2), *S*(4, 0), *T*(2, –2), and *U*(–5,0) is multiplied by $\frac{1}{3}$.

The area is multiplied by $1/3$.

The height of the rectangle is tripled. Describe the effect on the area.



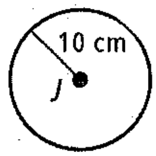
Area is tripled

The base and height of a rectangle with base 4 ft and height 5 ft are both doubled.

Perimeter is doubled

area is multiplied by 2^2 or 4

The radius of ⊙*J* is multiplied by 1/5.



Circumference is multiplied by $1/5$

Area is multiplied by $\left(\frac{1}{5}\right)^2$ or $\frac{1}{25}$

The base and height of the triangle with vertices *P*(2, 5), *Q*(2, 1), and *R*(7, 1) are tripled. Describe the effect on its area and perimeter.

Perimeter is tripled, and the area is multiplied by 9.

A circle has a circumference of 32π in. If the area is multiplied by 4, what happens to the radius?

It is doubled

An equilateral triangle has a perimeter of 21m. If the area is multiplied by $\frac{1}{2}$, what happens to the side length?

multiplied by $\frac{\sqrt{2}}{2}$

Effects of Changing Dimensions Proportionally		
Change in Dimensions	**Perimeter or Circumference**	**Area**
All dimensions multiplied by *a*	Changes by a factor of *a*	Changes by a factor of a^2

HW: Worksheet