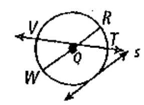
Teacher Copy

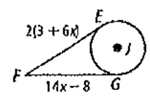
11.1 Lines That Intersect Circles

Learning Goal: Students will identify tangents, secants, and chords and use the properties to solve problems.

End – In – Mind



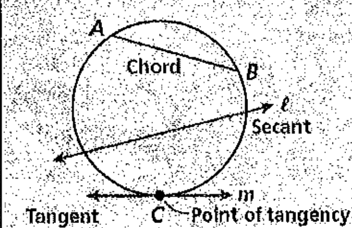
1. Identify each line or segment that intersects ⊙Q
2. Mount Mitchell peaks at 6,684 feet. What is the distance from this peak to the horizon, rounded to the nearest mile?



3. *FE* and *FG* are tangent to ⊙*F*. Find *F*.

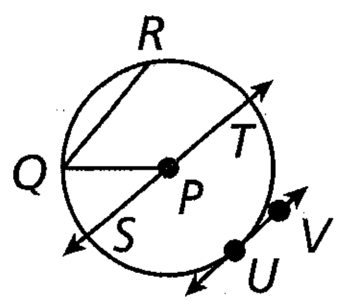
The Interior of a Circle **is the set of all points inside the circle.**
The Exterior of a Circle **is the set of all points outside the circle.**

Lines and Segments That Intersect Circles

TERM	DIAGRAM
A **chord** is a segment whose endpoints lie on a circle.	
A **secant** is a line that intersects a circle at two points.	
A **tangent** is a line in the same plane as a circle that intersects it at exactly one point.	
The point where the tangent and a circle intersect is called the **point of tangency**.	

Example 1:

chords: $\overline{QR}$, $\overline{ST}$
secant: $\overleftrightarrow{ST}$
tangent: $\overleftrightarrow{UV}$
diameter: $\overline{ST}$
radii: $\overline{PQ}$, $\overline{PT}$, $\overline{PS}$



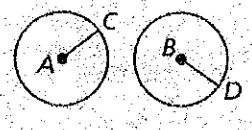
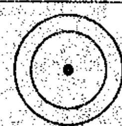
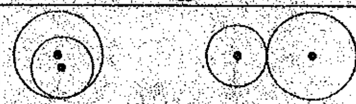
Quick Write: Name one difference between…

a. a chord and a secant:____________________

b. a chord and a diameter:____________________

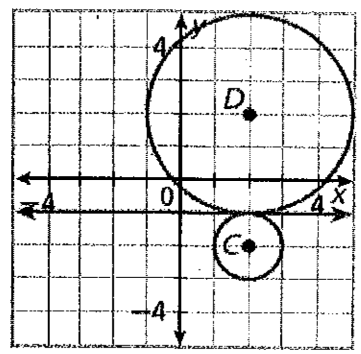
c. a secant and a tangent:____________________

Pairs of Circles

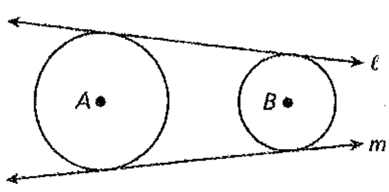
TERM	DIAGRAM
Two circles are **congruent circles** if and only if they have congruent radii.	 $\odot A \cong \odot B$ if $\overline{AC} \cong \overline{BD}$. $\overline{AC} \cong \overline{BD}$ if $\odot A \cong \odot B$.
Concentric circles are coplanar circles with the same center.	
Two coplanar circles that intersect at exactly one point are called **tangent circles**.	 Internally tangent circles Externally tangent circles

Example 2:
Find the length of each radius. Identify the point of tangency and write the equation of the tangent line at this point.

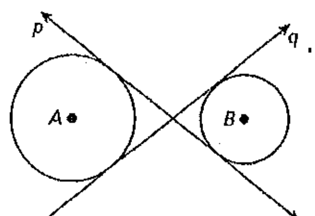
radius of $\odot C$: 1 radius of $\odot D$: 3 equation of tangent line: y = -1



A Common Tangent is a line that is tangent to two circles.

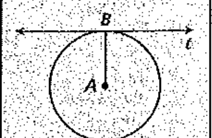
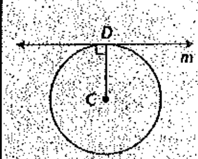


Lines ℓ and m are common external tangents to $\odot A$ and $\odot B$.



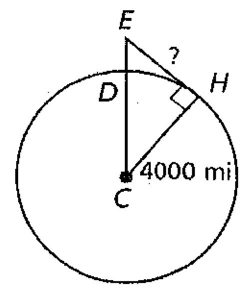
Lines p and q are common internal tangents to $\odot A$ and $\odot B$.

Theorems

THEOREM	HYPOTHESIS	CONCLUSION
11-1-1 If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency. (line tangent to ⊙ → line ⊥ to radius)	 ℓ is tangent to ⊙A	$\ell \perp \overline{AB}$
11-1-2 If a line is perpendicular to a radius of a circle at a point on the circle, then the line is tangent to the circle. (line ⊥ to radius → line tangent to ⊙)	 m is ⊥ to $\overline{CD}$ at D	m is tangent to ⊙C.

Example 3:

Early in its flight, the Apollo 11 spacecraft orbited Earth at an altitude of 120 miles. What was the distance from the spacecraft to Earth's horizon rounded to the nearest mile?



$EC = CD + ED$

$EC = 4000 + 120$

$EC = 4120$

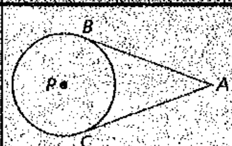
$EC^2 = CH^2 + EH^2$

$4120^2 = 4000^2 + EH^2$

$\sqrt{974400} = \sqrt{EH^2}$

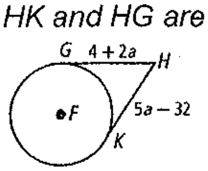
$EH \approx 987$ mi

Theorem 11-1-3

THEOREM	HYPOTHESIS	CONCLUSION
If two segments are tangent to a circle from the same external point, then the segments are congruent. (2 segs. tangent to ⊙ from same ext. pt. → segs. ≅)	 $\overline{AB}$ and $\overline{AC}$ are tangent to ⊙P.	$\overline{AB} \cong \overline{AC}$

Example 4:

HK and HG are tangent to ⊙F. Find HG.



$4 + 2a = 5a - 32$

$+32\ -2a\quad -2a\ +32$

$\frac{36}{3} = \frac{3a}{3}$

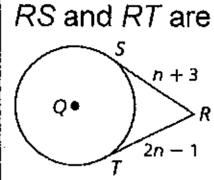
$a = 12$

$HG = 4 + 2(12)$

$HG = 48$

Example 5:

RS and *RT* are tangent to ⊙Q. Find *RS*.



$n + 3 = 2n - 1$

$-n\ +1\quad -n\ +1$

$4 = n$

$RS = 4 + 3$

$RS = 7$

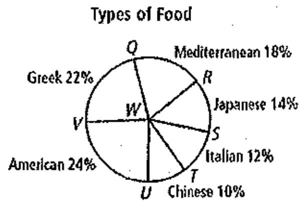
Homework: Worksheet HW 89

11.2 Arcs and Chords

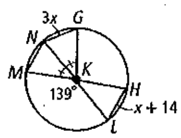
Learning Goal: Students will apply properties of arcs and chords.

End – In – Mind

1. The circle graph shows the types of cuisine available in a city. Find m arc TRQ.

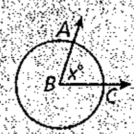
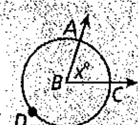


2. Find the measure of arc NGH
3. Find HL



A Central Angle is an angle whose vertex is the center of a circle. An Arc is an unbroken part of a circle consisting of two points called the endpoints and all the points on the circle between them.

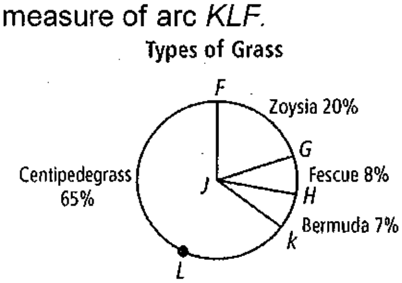
Arcs and Their Measure

ARC	MEASURE	DIAGRAM
A **minor arc** is an arc whose points are on or in the interior of a central angle.	The measure of a minor arc is equal to the measure of its central angle. $m\widehat{AC} = m\angle ABC = x°$	
A **major arc** is an arc whose points are on or in the exterior of a central angle.	The measure of a major arc is equal to 360° minus the measure of its central angle. $m\widehat{ADC} = 360° - m\angle ABC = 360° - x°$	
If the endpoints of an arc lie on a diameter, the arc is a **semicircle**.	The measure of a semicircle is equal to 180°. $m\widehat{EFG} = 180°$	

Minor arcs MAY be named by two points.
Major arcs and semicircles MUST be named by three points.

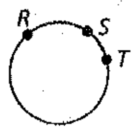
Example 1: Data Application

The circle graph shows the types of grass planted in the yards of one neighborhood. Find the measure of arc *KLF*.



$\widehat{KLF} = 65\% \cdot 360 = 234°$

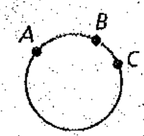
Adjacent Arcs are arcs of the same circle that intersect at exactly one point. Arc *RS* and arc *ST* are adjacent arcs.

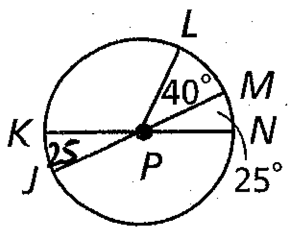
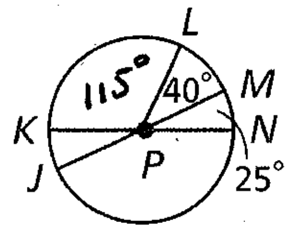


Postulate 11-2-1 **Arc Addition Postulate**

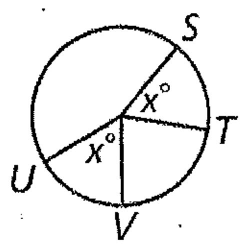
The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

$m\widehat{ABC} = m\widehat{AB} + m\widehat{BC}$

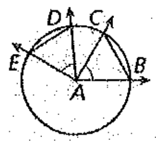
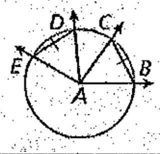
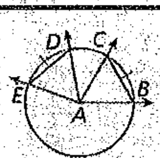


Example 2: m arc *JKL* = 140°	Example 3: m arc LJN = 295°
 180 −40 = 140	 180 −65 = 115 115+180 = 295°

Within a circle or congruent circles, ≅ arcs are two arcs that have the same measure. In the figure arc $ST \cong$ arc UV.



Theorem 11-2-2

THEOREM	HYPOTHESIS		CONCLUSION
In a circle or congruent circles: (1) Congruent central angles have congruent chords.		$\angle EAD \cong \angle BAC$	$\overline{DE} \cong \overline{BC}$
(2) Congruent chords have congruent arcs.		$\overline{ED} \cong \overline{BC}$	$\widehat{DE} \cong \widehat{BC}$
(3) Congruent arcs have congruent central angles.		$\widehat{ED} \cong \widehat{BC}$	$\angle DAE \cong \angle BAC$

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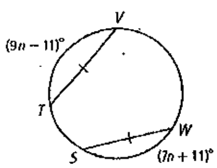
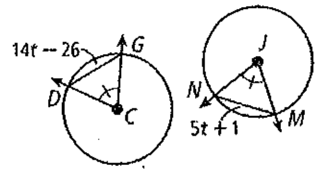
Example 3A:
$TV \cong WS$. Find m arc WS.



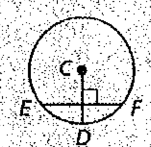
$9n-11=7n+11$
$-7n+11 \quad -7n+11$
$\frac{2n}{2}=\frac{22}{2}$
$n=11$
$WS=7(11)+11$
$WS=88°$

Example 3B:
$\odot C \cong \odot J$, and $m\angle GCD \cong m\angle NJM$. Find NM.

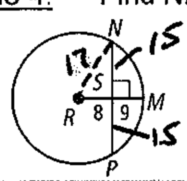


$14t-26=5t+1$
$-5t+26 \quad -5t+26$
$\frac{9t}{9}=\frac{27}{9}$
$t=3$
$NM=5(3)+1$
$NM=16°$

Theorems

THEOREM		HYPOTHESIS	CONCLUSION
11-2-3	In a circle, if a radius (or diameter) is perpendicular to a chord, then it bisects the chord and its arc.	 $\overline{CD} \perp \overline{EF}$	$\overline{CD}$ bisects $\overline{EF}$ and $\widehat{EF}$.
11-2-4	In a circle, the perpendicular bisector of a chord is a radius (or diameter).	 $\overline{JK}$ is $\perp$ bisector of $\overline{GH}$.	$\overline{JK}$ is a diameter of $\odot A$.

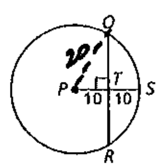
Example 4: Find NP.



$8^2+b^2=17^2$
$\sqrt{b^2}=\sqrt{225}$
$b=15$
$NP=15+15=30$

Homework: Worksheet
HW90

Check–It–Out Example 4: Find QR



$10^2+b^2=20^2$
$\sqrt{b^2}=\sqrt{300}$
$b=17.32$
$QR=17.32+17.32=34.6$

11.3 Sector Area and Arc Length

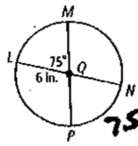
Learning Goal: Students will find the area of sectors and arc lengths.

End-In-Mind

Find each measure. Give answers in terms of π and rounded to the nearest hundredth.

1. area of sector *LQM*

2. length of arc *NP*



75°

$A = \pi r^2\left(\frac{75}{360}\right)$

$\pi 6^2\left(\frac{75}{360}\right)$

$A = 7.5\pi$° LQM or 23.56°

3. The gear of a grandfather clock has a radius of 3 in. To the nearest tenth of an inch, what *what distance does the gear cover when it rotates through an angle of 88°?*

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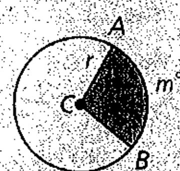
The area of a sector is a fraction of the circle containing the sector. To find the area of a sector whose central angle measures $m°$, multiply the area of the circle by

$$\frac{m°}{360°}.$$

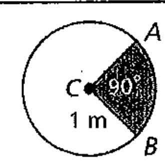
Sector of a Circle

TERM	NAME	DIAGRAM	AREA
A **sector of a circle** is a region bounded by two radii of the circle and their intercepted arc.	sector *ACB*		$A = \pi r^2\left(\frac{m°}{360°}\right)$

Write the degree symbol after m in the formula to help you remember to use degree measure not arc length.

Find the area of each sector. Give answers in terms of π and rounded to the nearest hundredth.

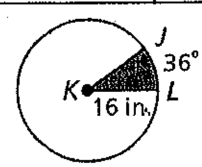
Check it out Example 1A: sector ACB



$\pi 1^2\left(\frac{90}{360}\right)$

0.25π or 0.79^2

Check it out Example 1B: sector JKL



$\pi 16^2\left(\frac{36}{360}\right)$

25.6π or 80.42 in^2

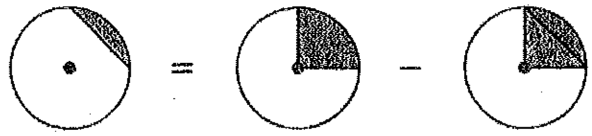
Example 2: A windshield wiper blade is 18 inches long. To the nearest square inch, what is the area covered by the blade as it rotates through an angle of 122°?

$\pi\ 18^2\left(\frac{122}{360}\right) = 109.8\pi$ or 344.95

345 in^2

A <u>Segment of a Circle</u> is a region bounded by an arc and its chord.

Area of a Segment



area of segment = area of sector − area of triangle

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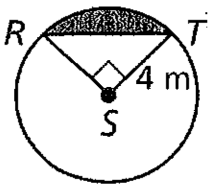
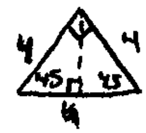
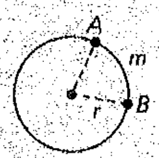
Check it out Example 3: **Find the area of segment *RST* to the nearest hundredth.**



Area of Sector	Area of Triangle	Area of Segment
$A = \pi r^2\left(\frac{m^\circ}{360}\right)$ $= 4^2\left(\frac{90}{360}\right)\pi$ $= 4\pi$ or 12.57	$A = \frac{1}{2}bh$  $A = \frac{1}{2}(4)(4)$ $A = 8$	$4\pi - 8 = 4.57\ m^2$

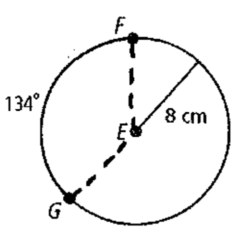
In the same way that the area of a sector is a fraction of the area of the circle, the length of an arc is a fraction of the circumference of the circle.

Arc Length

TERM	DIAGRAM	LENGTH
Arc length is the distance along an arc measured in linear units.		$L = 2\pi r\left(\frac{m^\circ}{360^\circ}\right)$

Find each arc length. Give answers in terms of π and rounded to the nearest hundredth.

Example 4A: Find Arc FG



$L = 2\pi r\left(\frac{m^\circ}{360}\right)$

$= 2(8)\left(\frac{134}{360}\right)\pi$

$L = 5.96\pi$ or 18.71 cm

Example 4B: an arc with measure 62° in a circle with radius 2 m

$L = 2\pi r\left(\frac{m^\circ}{360}\right)$

$= 2(2)\left(\frac{62}{360}\right)\pi$

$L = 0.69\pi$ or 2.17 m

Homework: Worksheet

HW 91

11.4 Inscribed Angles

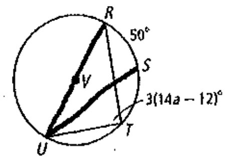
Learning Goal: Students will find the measure of an inscribed angle and use the properties to solve problems.

End-In-Mind

Find:

1. $\angle RUS$ = $25°$

$\frac{1}{2}(50)$



2. a

$3(14a-12) = \frac{1}{2}(180)$

$42a - 36 = 90$

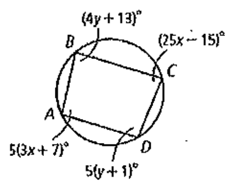
$+36 \quad +36$

$42a = 126$

$a = 3$

3. Find the angle measures of *ABCD*.

A: 95°
B: 85°
C: 85°
D: 95°



$4y + 13 + 5y + 5 = 180$

$9y + 18 = 180$

$-18 \quad -18$

$9y = 162$

$y = 18$

$15x + 35 + 25x - 15 = 180$

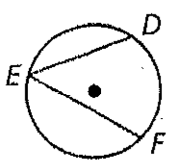
$40x + 20 = 180$

$-20 \quad -20$

$40x = 160$

$x = 4$

An __Inscribed Angle__ is an angle whose vertex is on a circle and whose sides contain chords of the circle. An __intercepted arc__ consists of endpoints that lie on the sides of an inscribed angle and all the points of the circle between them. A chord or arc __subtends__ an angle if its endpoints lie on the sides of the angle.



$\angle DEF$ is an inscribed angle.

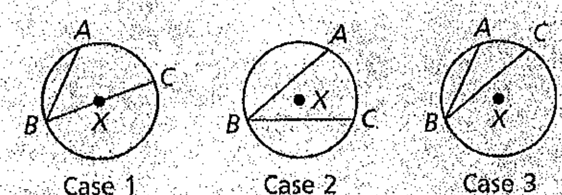
$\overparen{DF}$ is the intercepted arc.

$\overparen{DF}$ subtends $\angle DEF$.

Theorem 11-4-1 Inscribed Angle Theorem

The measure of an inscribed angle is half the measure of its intercepted arc.

$m\angle ABC = \frac{1}{2} m\overparen{AC}$



Quickwrite: Describe the difference between a Central Angle and an Inscribed Angle.

___

___

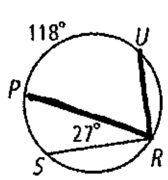
___

Find each measure:

Example 1a:

$m\angle PRU$ = $\frac{1}{2}(118) = 59°$

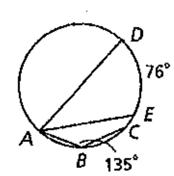
$m\overparen{SP}$ = $27 = \frac{1}{2} m\overparen{SP} = 54°$



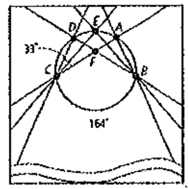
Check It Out! Example 1a:

$m\overparen{ADC}$ = ~~135.2~~ 270°

$m\angle DAE$ = $\frac{1}{2}(76) = 38°$

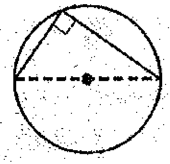


Example 2: **An art student turns in an abstract design for his art project. Find m∠*DFA*.**

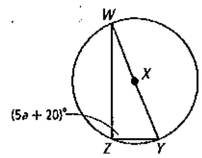


Theorem 11-4-3

An inscribed angle subtends a semicircle if and only if the angle is a right angle.



Example 3A: Find a

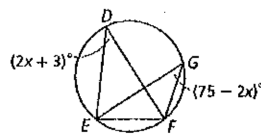


$5a + 20 = 90$

$\frac{5a}{5} = \frac{70}{5}$

$a = 14$

Check It Out! Example 3b: **Find m∠*EDF*.**



$2x + 3 = 75 - 2x$

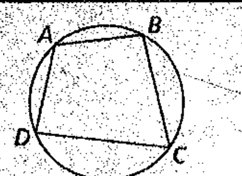
$2x \quad -3 \quad -3 \quad 2x$

$\frac{4x}{4} = \frac{72}{4}$ $x = 18$

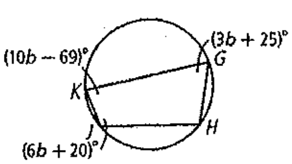
$m\angle EDF = 2(18) + 3$

$m\angle EDF = 39°$

Theorem 11-4-4

THEOREM	HYPOTHESIS	CONCLUSION
If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.	 *ABCD* is inscribed in ⊙*E*.	∠*A* and ∠*C* are supplementary. ∠*B* and ∠*D* are supplementary.

Example 4:
Find the angle measures of *GHJK*.



$\angle G + \angle J = 180$

$3b + 25 + 6b + 20 = 180$

$9b + 45 = 180$

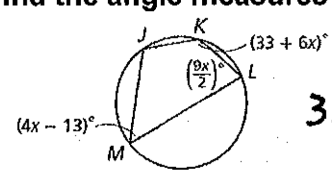
$-45 \quad -45$

$\frac{9b}{9} = \frac{135}{9}$

$b = 15$

K = 81°
J = 110°
G = 70°
H = 99°

Check It Out! Example 4:
Find the angle measures of *JKLM*.



$33 + 6x = 4x - 13$

K =
L =
M =
J =

Homework: Worksheet

HW92

11.5 Angle Relationships in Circles

Learning Goal: Students will find the measures of angles formed by lines that intersect circles.

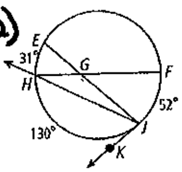
End-In-Mind

Find each measure:

1. m∠*FGJ*

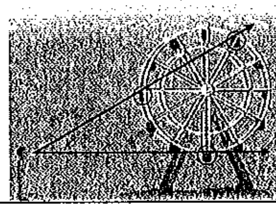
$\frac{1}{2}(31+52)$

41.5°



2. m∠*HJK* $\frac{1}{2}\cdot 130 = 65^\circ$

3. An observer watches people riding a Ferris wheel that has 12 equally spaced cars. Find *x*.



$\frac{1}{2}(3-5) = 1$

$\frac{1}{2}(150-90) =$

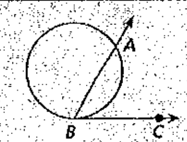
$x = 30^\circ$

$3\cdot 30 = 90$

$5\cdot 30 = 150$

$360 \div 12 = 30^\circ$

Theorem 11-5-1

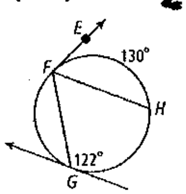
THEOREM	HYPOTHESIS	CONCLUSION
If a tangent and a secant (or chord) intersect on a circle at the point of tangency, then the measure of the angle formed is half the measure of its intercepted arc.	 Tangent $\overrightarrow{BC}$ and secant $\overrightarrow{BA}$ intersect at *B*.	$m\angle ABC = \frac{1}{2}m\overset{\frown}{AB}$

Find each measure:

Example 1:

m∠*EFH* $= \frac{1}{2}\cdot 130 = 65^\circ$

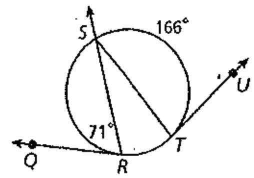
m (arc) GF $2\cdot 122 = 244$; $360 - 244 = 116^\circ$



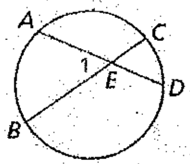
Check It Out! Example 1a:

m∠*STU* $\frac{1}{2}\cdot 166 = 83^\circ$

m (arc) SR $71\cdot 2 = 142^\circ$



Theorem 11-5-2

THEOREM	HYPOTHESIS	CONCLUSION
If two secants or chords intersect in the interior of a circle, then the measure of each angle formed is half the sum of the measures of its intercepted arcs.	 Chords $\overline{AD}$ and $\overline{BC}$ intersect at E.	$m\angle 1 = \frac{1}{2}(m\widehat{AB} + m\widehat{CD})$

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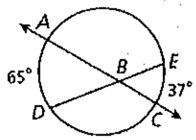
Find each angle measure:

Check it out Example 2A:

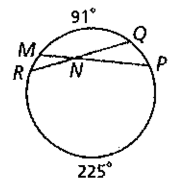
m∠ABD $= \frac{1}{2}(65+37)$

$= 51°$



Check it out Example 2B:

m∠RNM

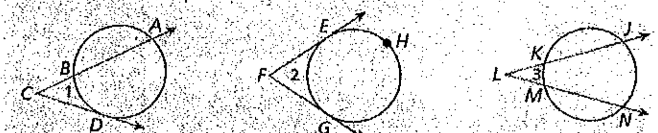


$\frac{1}{2}(91+225) = 158$

$180 - 158 = 22°$

Theorem 11-5-3

If a tangent and a secant, two tangents, or two secants intersect in the exterior of a circle, then the measure of the angle formed is half the difference of the measures of its intercepted arcs.

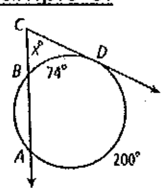


$m\angle 1 = \frac{1}{2}(m\widehat{AD} - m\widehat{BD})$ $m\angle 2 = \frac{1}{2}(m\widehat{EHG} - m\widehat{EG})$ $m\angle 3 = \frac{1}{2}(m\widehat{JN} - m\widehat{KM})$

Find the value of *x*.

Example 3:

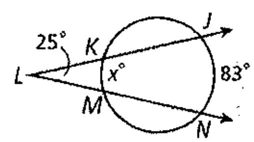
B.



$\frac{1}{2}(200-74)$

$63°$

Check it out Example 3:



$\frac{25}{\frac{1}{2}} = \frac{\frac{1}{2}(83-x)}{\frac{1}{2}}$

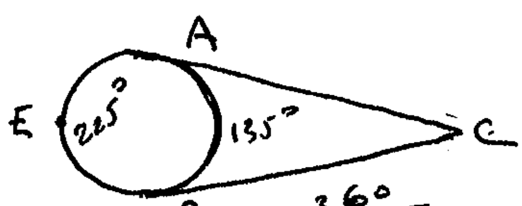
$50 = 83 - x$

$-83 \quad -83$

$-x = -33$

$x = 33$

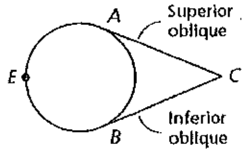
Check It Out! Example 4: **Two of the six muscles that control eye movement are attached to the eyeball and intersect behind the eye. If m *AEB* = 225°, what is m∠*ACB*?**



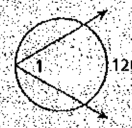
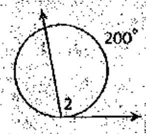
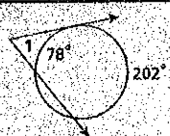
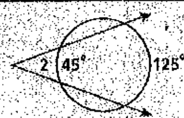
$360 - 225 = 135$

$m\angle ACB = \frac{1}{2}(225-135)$

$m\angle ACB = 45°$



Angle Relationships in Circles

VERTEX OF THE ANGLE	MEASURE OF ANGLE	DIAGRAMS	
On a circle	Half the measure of its intercepted arc	 $m\angle 1 = 60°$	 $m\angle 2 = 100°$
Inside a circle	Half the sum of the measures of its intercepted arcs		$m\angle 1 = \frac{1}{2}(44° + 86°) = 65°$
Outside a circle	Half the difference of the measures of its intercepted arcs	 $m\angle 1 = \frac{1}{2}(202° - 78°) = 62°$	 $m\angle 2 = \frac{1}{2}(125° - 45°) = 40°$

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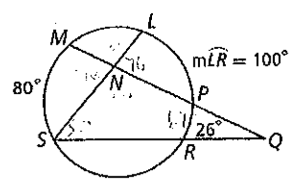
Check It Out! Example 5: **Find m (arc) *LP***



Homework: Worksheet

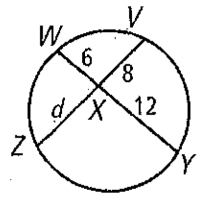
Hw 93

11.6 Segment Relationships in Circles

Learning Goal: Students will find the lengths of segments formed by lines that intersect circles.

End-In-Mind

1. Find the value of *d* and the length of each chord.

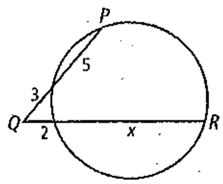


$12 \cdot 6 = d \cdot 8$

$\frac{72}{8} = \frac{d \cdot 8}{8}$

$d = 9$

2. Find the value of *x* and the length of each secant segment.

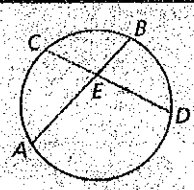


$8 \cdot 3 = 2x \cdot 2$

$\frac{24}{2} = \frac{2x \cdot 2}{2}$

$\frac{2x}{2} = \frac{12}{2}$ $x = 6$

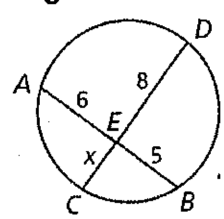
Theorem 11-6-1 **Chord-Chord Product Theorem**

THEOREM	HYPOTHESIS	CONCLUSION
If two chords intersect in the interior of a circle, then the products of the lengths of the segments of the chords are equal.	 Chords $\overline{AB}$ and $\overline{CD}$ intersect at *E*.	$AE \cdot EB = CE \cdot ED$

Check It Out! Example 1: **Find the value of *x* and the length of each chord.**

$6 \cdot 5 = 8 \cdot x$

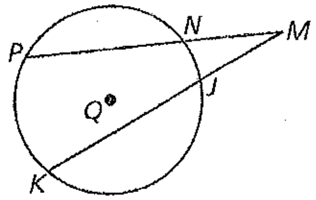
$\frac{30}{8} = \frac{8 \cdot x}{8}$ $x = 3.75$



AB = 11

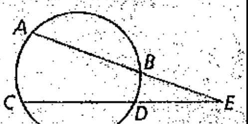
DC = 11.75

A Secant Segment is a segment of a secant with at least one endpoint on the circle. An external Secant Segment is a secant segment that lies in the exterior of the circle with one endpoint on the circle.

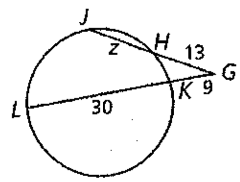


$\overline{PM}$, $\overline{NM}$, $\overline{KM}$, and $\overline{JM}$ are secant segments of $\odot Q$. $\overline{NM}$ and $\overline{JM}$ are external secant segments.

Theorem 11-6-2 Secant-Secant Product Theorem

THEOREM	HYPOTHESIS	CONCLUSION
If two secants intersect in the exterior of a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment. (whole • outside = whole • outside)	 Secants $\overline{AE}$ and $\overline{CE}$ intersect at E.	$AE \cdot BE = CE \cdot DE$

Check It Out! Example 3: **Find the value of *z* and the length of each secant segment.**



$JG \cdot HG = LG \cdot KG$

$(13+z)13 = 39 \cdot 9$

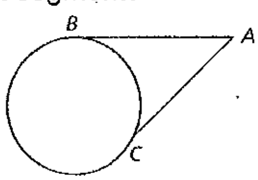
$169 + 13z = 351$

$-169 \qquad -169$

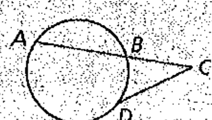
$13z = 182 \qquad z = 14$

$JG = 27 \qquad LG = 39$

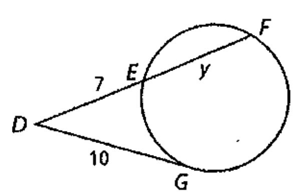
A Tangent Segment is a segment of a tangent with one endpoint on the circle. ***AB*** and ***AC*** are tangent segments.



Theorem 11-6-3 Secant-Tangent Product Theorem

THEOREM	HYPOTHESIS	CONCLUSION
If a secant and a tangent intersect in the exterior of a circle, then the product of the lengths of the secant segment and its external segment equals the length of the tangent segment squared. (whole • outside = tangent²)	 Secant $\overline{AC}$ and tangent $\overline{DC}$ intersect at C.	$AC \cdot BC = DC^2$

Check It Out! Example 4: **Find the value of *y*.**



$(7+y)7 = 10^2$

$49 + 7y = 100$

$-49 \qquad -49$

$\frac{7y}{7} = \frac{51}{7} \qquad y = 7.29$

Homework Worksheet

Hw 94