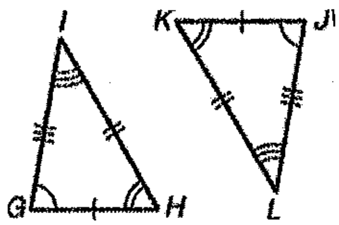
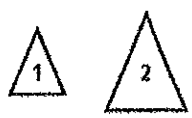
7.2 Ratios in Similar Polygons

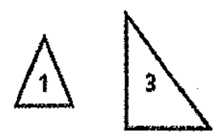
Learning Goals: Students will identify similar polygons and apply properties of similar polygons.
Name the corresponding angles and sides in the figures below.



Figures that are similar (~) have the Same Shape but not necessarily the Same Size.

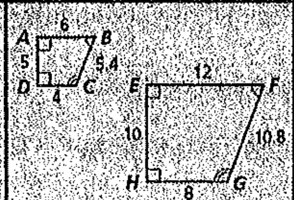


$\triangle 1$ is similar to $\triangle 2$ ($\triangle 1 \sim \triangle 2$).

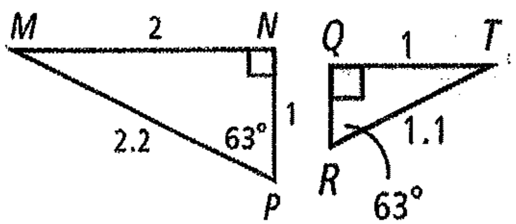
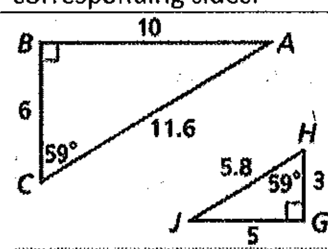


$\triangle 1$ is not similar to $\triangle 3$ ($\triangle 1 \nsim \triangle 3$).

Similar Polygons

DEFINITION	DIAGRAM	STATEMENTS
Two polygons are **similar polygons** if and only if their corresponding angles are congruent and their corresponding side lengths are proportional.	 ABCD ~ EFGH	$\angle A \cong \angle E$ $\angle B \cong \angle F$ $\angle C \cong \angle G$ $\angle D \cong \angle H$ $\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE} = \frac{1}{2}$

Example 1: Describing Similar Polygons

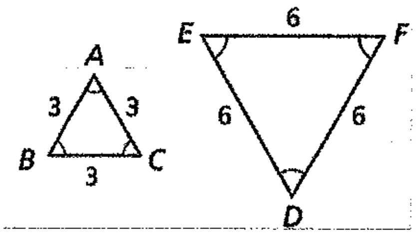
A. Identify the pairs of congruent angles and corresponding sides.	B. Identify the pairs of congruent angles and corresponding sides.
	
Angles: $\angle M \cong \angle T$, $\angle N \cong \angle Q$, $\angle P \cong \angle R$ Sides: $\frac{MN}{TQ} = \frac{NP}{QR} = \frac{MP}{TR} = \frac{2}{1}$	Angles: $\angle A \cong \angle J$, $\angle B \cong \angle G$, $\angle C \cong \angle H$ Sides: $\frac{AB}{JG} = \frac{BC}{GH} = \frac{AC}{JH} = 2$

Explain how you use the congruent angles to help you determine the corresponding sides?

A Similarity ratio is the ratio of the lengths of the corresponding sides of two similar polygons. The similarity ratio of ΔABC to ΔDEF is $\frac{3}{6}$, or $\frac{1}{2}$. The similarity ratio of ΔDEF to ΔABC is $\frac{6}{3}$, or 2.

Writing Math

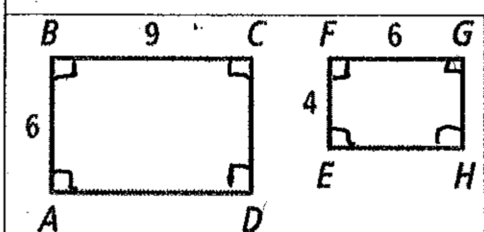
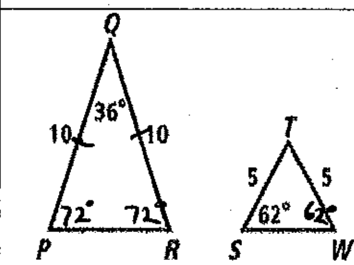
Writing a similarity statement is like writing a congruence statement—be sure to list corresponding vertices in the same order.



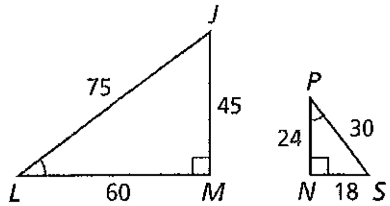
Example 2: Identifying Similar Polygons

Determine whether the polygons are similar. If so, write the similarity ratio and a similarity statement.

∠s have to be ≅ for the figures to be ~

A. rectangles ABCD and EFGH	B. ΔPQR and ΔSTW
	
Yes ; $\frac{3}{2}$ rect ABCD ~ rect EFGH $\frac{9}{6} = \frac{3}{2}$ $\frac{BC}{FG}$ $\frac{6}{4} = \frac{3}{2}$ $\frac{BA}{FE}$	No ∠'s are not ≅

C. Determine if ΔJLM ~ ΔNPS. If so, write the similarity ratio and a similarity statement.



$\frac{45}{18} = \frac{5}{2}$ $\frac{60}{24} = \frac{5}{2}$ $\frac{75}{30} = \frac{5}{2}$

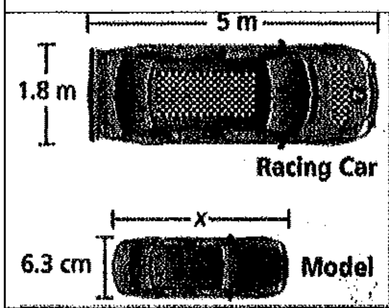
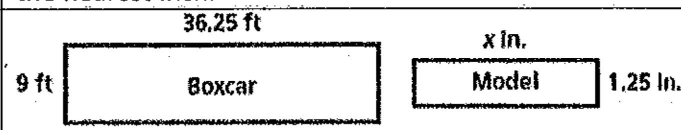
Yes, $\frac{5}{2}$, $\triangle LMJ \sim \triangle PNS$

What is the least amount of information necessary to determine that 2 isosceles triangles are similar?

____

____

Example 3: Application

A. Find the length of the model to the nearest tenth of a centimeter.	**B.** A boxcar has the dimensions shown. A model of the boxcar is 1.25 in. wide. Find the length of the model to the nearest inch.
	
$\frac{6.3}{1.8} = \frac{x}{5} = 17.5\text{cm}$ $6.5(5) = 1.8(x)$	$\frac{1.25}{9} = \frac{x}{36.25}$ $(36.25)1.25 = 9x$ $x = 5.03$ in.

What similarity ratio do congruent figures have? 1:1

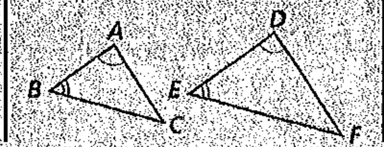
Similar Polygons	
Definition	Similarity Statement
Examples	Non-Examples

Homework: Page 465, # 7-17, 19, 20

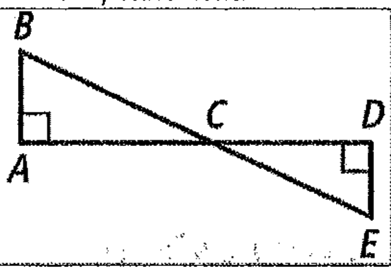
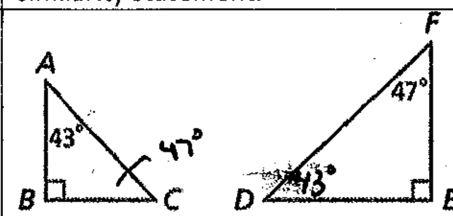
7.3 Triangle Similarity: AA, SSS, SAS

Learning Goals: Students will prove certain triangles are similar by using AA, SSS, and SAS and use triangle similarity to solve problems.

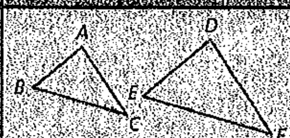
Postulate 7-3-1 Angle-Angle (AA) Similarity

POSTULATE	HYPOTHESIS	CONCLUSION
If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.		$\triangle ABC \sim \triangle DEF$

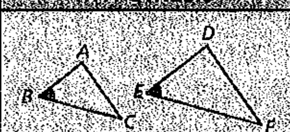
Example 1: Using the AA Similarity Postulate

A. Explain why the triangles are similar and write a similarity statement.	B. Explain why the triangles are similar and write a similarity statement.
	
$\angle A \cong \angle D$ Rt.∠≅ Thm. $\angle ACB \cong \angle DCE$ Vert Ag. Thm. $\triangle ACB \sim \triangle DEC$ by AA~	$\angle C \cong \angle F$ $\angle B \cong \angle E$ Rt.∠≅ Thm. $\triangle ABC \sim \triangle DEF$ by AA~

Theorem 7-3-2 Side-Side-Side (SSS) Similarity

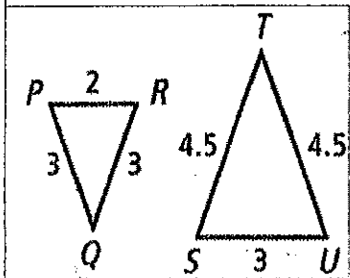
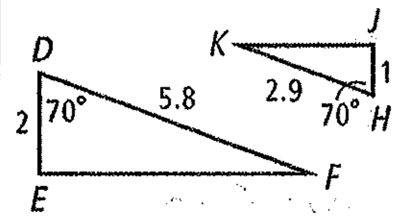
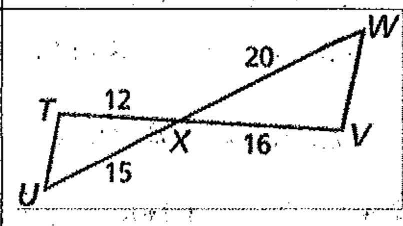
THEOREM	HYPOTHESIS	CONCLUSION
If the three sides of one triangle are proportional to the three corresponding sides of another triangle, then the triangles are similar.		$\triangle ABC \sim \triangle DEF$

Theorem 7-3-3 Side-Angle-Side (SAS) Similarity

THEOREM	HYPOTHESIS	CONCLUSION
If two sides of one triangle are proportional to two sides of another triangle and their included angles are congruent, then the triangles are similar.	 $\angle B \cong \angle E$	$\triangle ABC \sim \triangle DEF$

Example 2: Verifying Triangle Similarity

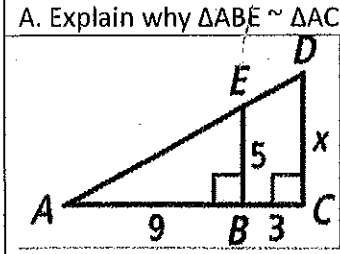
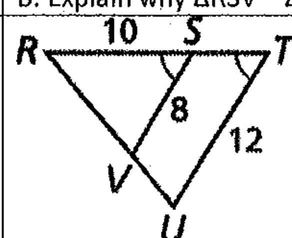
Verify that the triangles are similar.

A. ΔPQR and ΔSTU	B. ΔDEF and ΔHJK	C. ΔTXU ~ ΔVXW
		
$\frac{PQ}{ST}=\frac{QR}{TU}=\frac{PR}{SU}=\frac{2}{3}$ ΔPQR~ΔSTU by SSS~	m∠D = m∠H = 70° ∠D ≅ ∠H $\frac{DE}{JH}=\frac{2}{1}$ $\frac{DF}{HK}=\frac{5.8}{2.9}=\frac{2}{1}$ Δ DEF ~ Δ HJK by SAS~	∠TXU ≅ ∠VXW VA Thm. $\frac{TX}{VX}=\frac{12}{16}=\frac{3}{4}$ $\frac{XU}{XW}=\frac{15}{20}=\frac{3}{4}$ ΔTXU ~ ΔVXW SAS~

Explain why ASA and AAS are NOT similarity theorems.

Can you use just 2 pairs of corresponding sides when proving triangles similar?

Example 3: Finding Lengths in Similar Triangles

A. Explain why ΔABE ~ ΔACD, and then find CD.	B. Explain why ΔRSV ~ ΔRTU and then find RT.
	 RT = x $\frac{SV}{TU}=\frac{RS}{RT}$ $\frac{8}{12}=\frac{10}{x}$ $\frac{8x}{8}=\frac{120}{8}$ x = 15
∠A ≅ ∠A Ref. Prop ∠ABE ≅ ∠ACD Rt ∠s ≅ Δ ABE ~ Δ ACD by AA~ CD = $\frac{60}{9}=\frac{20}{3}$ or $6\frac{2}{3}$	∠RSV ≅ ∠T Ref Prop. ≅ ∠R ≅ ∠R Δ RSV ~ Δ RTU by AA~ RT = 15

Example 5: Engineering Application

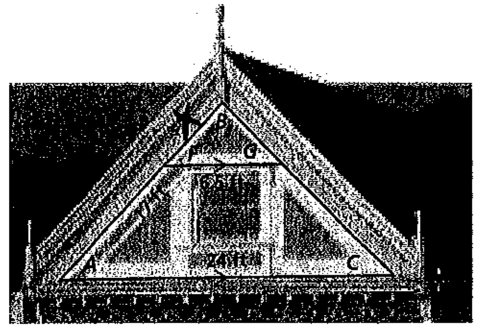
The photo shows a gable roof. AC || FG, ΔABC ~ ΔFBG. Find BA to the nearest tenth of a foot.

$\overline{AC} \parallel \overline{FG}$ Given

$\angle BFG \cong \angle BAC$ Corr ∠s Thm.

$\angle B \cong \angle B$ Refl. Prop. ≅

$\triangle ABC \sim \triangle FBG$ by AA ~



$\frac{BA}{BF} = \frac{AC}{FG}$ $\frac{17+x}{x} = \frac{24}{6.5}$ $24x = 6.5(17+x)$

$24x = \cancel{6.5x} + 110.5$
$-6.5x \quad -6.5x$

$\frac{17.5x}{17.5} = \frac{110.5}{17.5}$ $x = 6.3$

You learned in Chapter 2 that the Reflexive, Symmetric, and Transitive Properties of Equality have corresponding properties of congruence. These properties also hold true for similarity of triangles.

BA = 17 + 6.3 = 23.3 ft.

Properties of Similarity

Reflexive Property of Similarity

$\triangle ABC \sim \triangle ABC$ (Reflex. Prop. of ~)

Symmetric Property of Similarity

If $\triangle ABC \sim \triangle DEF$, then $\triangle DEF \sim \triangle ABC$. (Sym. Prop. of ~)

Transitive Property of Similarity

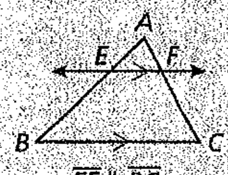
If $\triangle ABC \sim \triangle DEF$ and $\triangle DEF \sim \triangle XYZ$, then $\triangle ABC \sim \triangle XYZ$. (Trans. Prop. of ~)

Homework: Page 475, 11-24

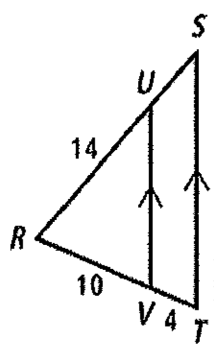
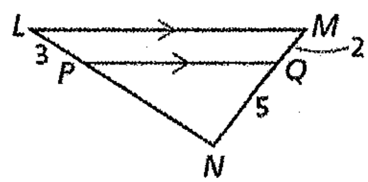
7.4 Applying Properties of Similar Triangles

Learning Goals: Students will use properties of similar triangles to find segment length and apply proportionality and triangle angle bisector theorems.

Theorem 7-4-1 Triangle Proportionality Theorem

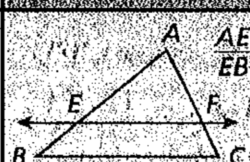
THEOREM	HYPOTHESIS	CONCLUSION
If a line parallel to a side of a triangle intersects the other two sides, then it divides those sides proportionally.	 $\overline{EF} \parallel \overline{BC}$	$\frac{AE}{EB} = \frac{AF}{FC}$

Example 1: Finding the Length of a Segment

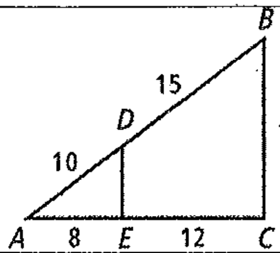
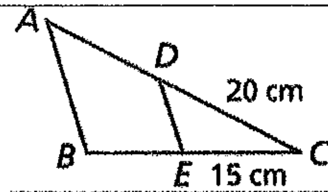
A. Find US.	B. Find PN.
It is given that $\overline{ST} \parallel \overline{UV}$, so $\frac{US}{RU} = \frac{VT}{RV}$ by the Triangle Proportionality Theorem.  $\frac{US}{14} = \frac{4}{10}$ $\frac{10(US)}{10} = \frac{56}{10}$ $US = \frac{28}{5}$ or $5\frac{3}{5}$	 $\frac{LP}{PN} = \frac{MQ}{NQ}$ $\frac{3}{PN} = \frac{2}{5}$ $\frac{2PN}{2} = \frac{15}{2}$ $PN = \frac{15}{2}$ or 7.5

What are two different ways you can set up proportions?

Theorem 7-4-2 Converse of the Triangle Proportionality Theorem

THEOREM	HYPOTHESIS	CONCLUSION
If a line divides two sides of a triangle proportionally, then it is parallel to the third side.	 $\frac{AE}{EB} = \frac{AF}{FC}$	$\overleftrightarrow{EF} \parallel \overline{BC}$

Example 2: Verifying Segments are Parallel

A. Verify that $\overline{DE} \parallel \overline{BC}$	B. AC = 36 cm, and BC = 27 cm. Verify that $\overline{DE} \parallel \overline{AB}$ by Conv. △ Prop. Thm.
 $\frac{AD}{DB} = \frac{10}{15} = \frac{2}{3}$ $\frac{AE}{EC} = \frac{8}{12} = \frac{2}{3}$ $\frac{AD}{DB} = \frac{AE}{EC}$ $\overline{DE} \parallel \overline{BC}$ by Conv. △ Prop. Thm.	 $\frac{DC}{AD} = \frac{EC}{BE}$ AD = 16 BE = 12 $\frac{DC}{AD} = \frac{20}{16} = \frac{5}{4}$ $\frac{EC}{BE} = \frac{15}{12} = \frac{5}{4}$

What other proportions can you use to verify that the segments are parallel?

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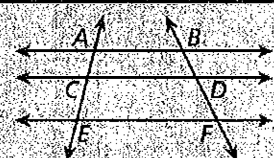
Corollary 7-4-3 **Two-Transversal Proportionality**

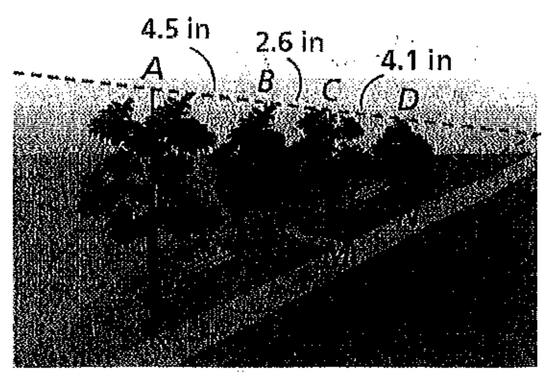
THEOREM	HYPOTHESIS	CONCLUSION
If three or more parallel lines intersect two transversals, then they divide the transversals proportionally.		$\frac{AC}{CE} = \frac{BD}{DF}$

Example 3: Art Application

Suppose that an artist decided to make a larger sketch of the trees. In the figure, if AB = 4.5 in., BC = 2.6 in., CD = 4.1 in., and KL = 4.9 in., find LM and MN to the nearest tenth of an inch.

$\frac{KL}{LN} = \frac{AB}{BD}$

BD = BC + CD 2.6 + 4.1 = 6.7



Use the diagram to find LM and MN to the nearest tenth.

BD = 6.7

$\frac{AB}{BC} = \frac{KL}{LM}$

$\frac{4.5}{2.6} = \frac{4.9}{x}$

$4.5x = 2.6(4.9)$

$\frac{4.5x}{4.5} = \frac{12.74}{4.5}$

LM~~x~~ = 2.8

$\frac{AB}{BD} = \frac{KL}{LN}$

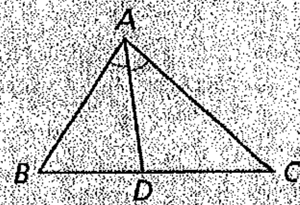
$\frac{4.5}{6.7} = \frac{4.9}{LN}$

$\frac{32.83}{4.5} = \frac{4.5LN}{4.5}$

7.3 − 2.8 ⇒ MN = 4.5

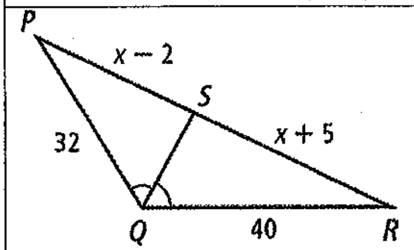
LN = 7.3

Theorem 7-4-4 **Triangle Angle Bisector Theorem**

THEOREM	HYPOTHESIS	CONCLUSION
An angle bisector of a triangle divides the opposite side into two segments whose lengths are proportional to the lengths of the other two sides. (△ ∠ Bisector Thm.)		$\frac{BD}{DC} = \frac{AB}{AC}$

Example 4: Using the Triangle Angle Bisector Theorem

A. Find PS and SR.



$\frac{SR}{PS} = \frac{QR}{PQ}$ $\quad \frac{x+5}{x-2} = \frac{40}{32}$

$40(x-2) = 32(x+5)$

$40x - 80 = 32x + 160$
$-32x + 80 \quad -32x + 80$

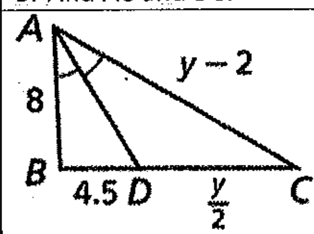
$\frac{8x}{8} = \frac{240}{8}$

$x = 30$

$PS = 30 - 2 = 28$

$SR = 30 + 5 = 35$

B. Find AC and DC.



$\frac{BD}{DC} = \frac{AB}{AC}$ $\quad \frac{4.5}{\frac{y}{2}} = \frac{8}{y-2}$

$4.5(y-2) = 8\left(\frac{y}{2}\right)$

$4.5y - 9 = \frac{8y}{2}$ or $4y$

$\frac{-9}{-.5} = \frac{-.5y}{-.5}$

$y = +18$

$AC = 18 - 2 = 16$

$DC = \frac{18}{2} = 9$

Homework: Page 485, 8-20

7.5 Using Proportional Relationships

Learning Goals: Students will use ratios to make indirect measurements and use scale drawings to solve problems.

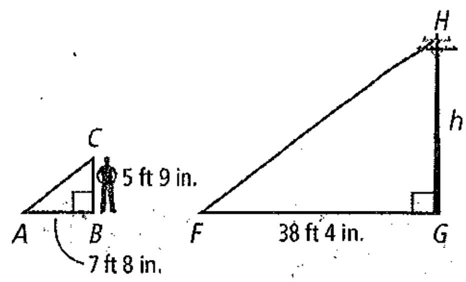
Indirect measurement is any method that uses formulas, similar figures, and/or proportions to measure an object.

Helpful Hint

Whenever dimensions are given in both feet and inches, you must convert them to either feet or inches before doing any calculations.

Example 1: Measurement Application

Tyler wants to find the height of a telephone pole. He measured the pole's shadow and his own shadow and then made a diagram. What is the height h of the pole?



Step 1: Identify corresponding lengths

$$\frac{CB}{HG} = \frac{AB}{FG}$$

Step 2: Convert all measurements to consistent units

$$\frac{69}{h} = \frac{92}{460} \qquad 92(h) = 460(69)$$

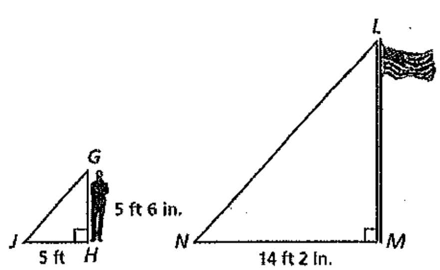
Step 3: Set up a proportion and solve.

$$\frac{92h}{92} = \frac{31740}{92} \qquad h = 345 \text{ in.}$$

28 ft. 9 in.

Check It Out! Example 1

A student who is 5 ft. 6 in. tall measured shadows to find the height *LM* of a flagpole. What is *LM*?



Step 1: $\frac{GH}{JH} = \frac{LM}{NM}$

Step 2: $\frac{66}{60} = \frac{LM}{170}$

Step 3: $\frac{60\,LM}{60} = \frac{11220}{60}$

$LM = 187$ 15ft 7in

Explain how a right triangle models the relationship between the shadow of an object and its height.

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A Scale Drawing represents an object as smaller than or larger than its actual size. The drawing's scale is the ratio of any length in the drawing to the corresponding actual length.

For example, on a map with a scale of 1 cm: 1500 m, one centimeter on the map represents 1500 m in actual distance.

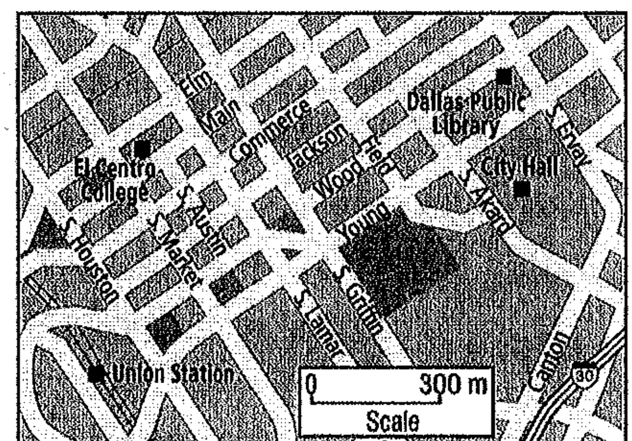
Remember!
A proportion may compare measurements that have different units.

Example 2: Solving for a Dimension

On a Wisconsin road map, Kristin measured a distance of $11\frac{1}{8}$ in. from Madison to Wausau. The scale of this map is 1inch: 13 miles. What is the actual distance between Madison and Wausau to the nearest mile?

$\frac{1}{13} = \frac{11\frac{1}{8}}{x}$ $x = 145$ mi

Example 3



Find the actual distance between City Hall and El Centro College.

$\frac{1.5}{300} = \frac{5}{x}$ $\frac{1.5x}{1.5} = \frac{1500}{1.5}$

$x = 900$ m

Find the actual distance between Dallas Public Library and Union Station.

$\frac{1.5}{300} = \frac{6.5}{x}$ $\frac{1.5x}{1.5} = \frac{1950}{1.5}$

$x = 1300$ m

Example 4: Making a Scale Drawing

Lady Liberty holds a tablet in her left hand. The tablet is 7.19 m long and 4.14 m wide. If you made a scale drawing using the scale 1 cm: 0.75 m, what would be the dimensions to the nearest tenth?

$\frac{1}{0.75} = \frac{x}{7.19}$ $\frac{.75x}{.75} = \frac{7.19}{.75}$ $x = 9.6$ cm l

$\frac{1}{.75} = \frac{x}{4.14}$ $\frac{4.14}{.75} = 5.5$ cm W

Example 5

The rectangular central chamber of the Lincoln Memorial is 74 ft. long and 60 ft. wide. Make a scale drawing of the floor of the chamber using a scale of 1 in: 20 ft.

$\frac{1}{20} = \frac{l}{74}$ $\frac{1}{20} = \frac{w}{60}$

$\frac{70}{20}$ = 3.5 in. long $\frac{60}{20} = 3$ in wide

How can you set up proportions using measures in different units? ______________________

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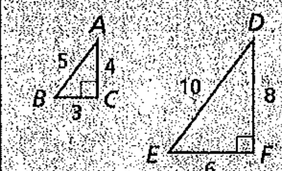
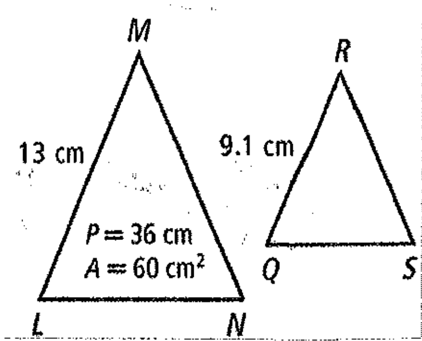
Similar Triangles **Similarity, Perimeter, and Area Ratios**

STATEMENT	RATIO
$\triangle ABC \sim \triangle DEF$ 	Similarity ratio: $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = \frac{1}{2}$ Perimeter ratio: $\frac{\text{perimeter } \triangle ABC}{\text{perimeter } \triangle DEF} = \frac{12}{24} = \frac{1}{2}$ Area ratio: $\frac{\text{area } \triangle ABC}{\text{area } \triangle DEF} = \frac{6}{24} = \frac{1}{4} = \left(\frac{1}{2}\right)^2$

Theorem 7-5-1 **Proportional Perimeters and Areas Theorem**

If the similarity ratio of two similar figures is $\frac{a}{b}$, then the ratio of their perimeters is $\frac{a}{b}$ and the ratio of their areas is $\frac{a^2}{b^2}$, or $\left(\frac{a}{b}\right)^2$.

Example 6: Using Ratios to Find Perimeters and Areas



Given that ΔLMN:ΔQRT, find the perimeter P and area A of ΔQRS.

The similarity ratio of ΔLMN to ΔQRS is

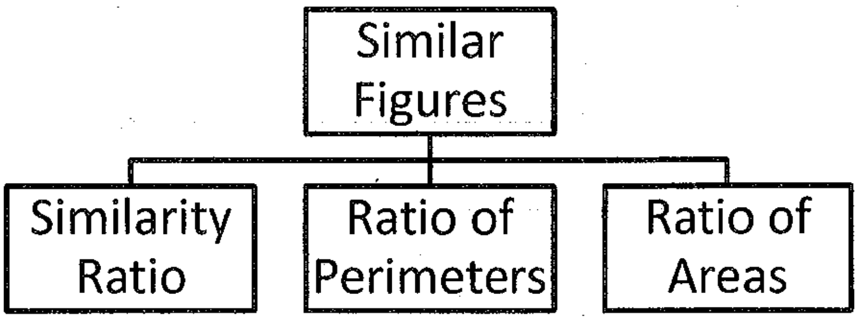
By the Proportional Perimeters and Areas Theorem, the ratio of the triangles' perimeters is 4/5, and the ratio of the triangles' areas is $(4/5)^2$ or 16/25.

Perimeter	Area
$\frac{36}{P} = \frac{4}{5}$ $4P = 5(36)$ $P = 45$ ft.	$\frac{48}{A} = \frac{16}{25}$ $16A = 25 \cdot 48$ $A = 75$ ft^2

Example 7

$\Delta ABC \sim \Delta DEF$, BC = 4 mm, and EF = 12 mm. If P = 42 mm and A = 96 mm^2 for ΔDEF, find the perimeter and area of ΔABC.

Similarity Ratio	Perimeter	Area
$\frac{EF}{BC} = \frac{12}{4}$ or $\frac{3}{1}$	$\frac{42}{P} = \frac{3}{1}$ $\frac{3p}{3} = \frac{42}{3}$ P = 14 mm	$\frac{96}{A} = \frac{3}{1}$ $\frac{3A}{3} = \frac{96}{3}$ A = 32 mm^2



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