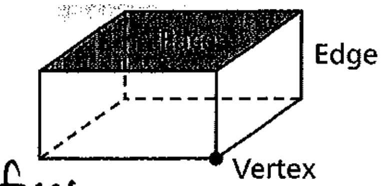
10.1 Solid Geometry

Learning Objectives: Students will classify three-dimensional figures according to their properties and use nets and cross sections to analyze three-dimensional figures.

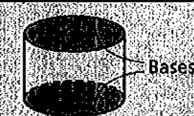
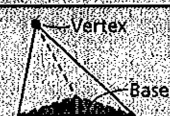
Face – Flat surface of the figure

Edge – Segment that is the intersection of two faces.

Vertex - the point that is the intersection of 3 or more faces.



Three-Dimensional Figures

TERM	EXAMPLE
A **prism** is formed by two parallel congruent polygonal faces called *bases* connected by faces that are parallelograms.	
A **cylinder** is formed by two parallel congruent circular bases and a curved surface that connects the bases.	
A **pyramid** is formed by a polygonal base and triangular faces that meet at a common vertex.	
A **cone** is formed by a circular base and a curved surface that connects the base to a vertex.	

A cube is a prism with six square faces. Draw two ways:

Prisms and pyramids are named for the shape of their bases.

Triangular prism

Rectangular prism

Pentagonal prism

Hexagonal prism

Triangular pyramid

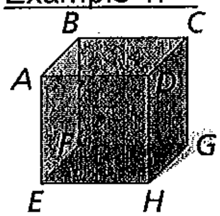
Rectangular pyramid

Pentagonal pyramid

Hexagonal pyramid

Classify the figure. Give an example of the following: vertices, edges, and bases.

Example 1:



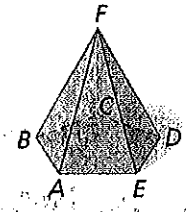
Classify: Rectangular Prism

Verticies: A,B,C,D,E,F,G,H

Edges: AB,BC,CD,AD,EF,FG
GH,HE,AE,BF,CG,DH

Bases: ABCD, EFGH, ABFE,
DCGH, ADHE, BCGF

Example 2:



Class: pentagonal pyramid

Vert: A, B, C, D, E, F

Edge: $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EA}$, $\overline{AF}$, $\overline{BF}$, $\overline{CF}$, $\overline{DF}$, $\overline{EF}$

Base: ABCDE

Example 3:

Class: Cone

Vert: N

Edge: None

Base: ⊙ M

Example 4:

Class: Triangular Prism

Vert: T, U, V, W, X

Edge: $\overline{TU}$, $\overline{UV}$, $\overline{VT}$, $\overline{WX}$, $\overline{XY}$, $\overline{WY}$, $\overline{TW}$, $\overline{XU}$, $\overline{VY}$

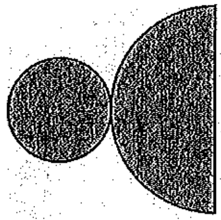
Base: △TUV, △WXY

A Net is a diagram of the surfaces of a three-dimensional figure that can be folded to form the three-dimensional figure.

**Hint *To identify a three-dimensional figure from a net, look at the number of faces and the shape of each face.*

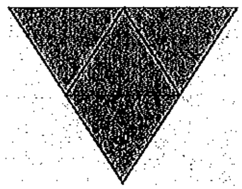
Name the three-dimensional figure that can be made from the given net.

Example 5:



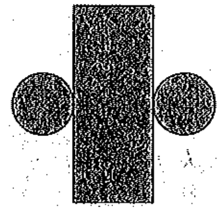
Cone

Example 6:



Triangular Pyramid

Example 7:

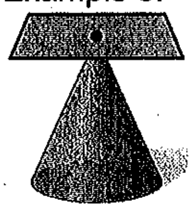


Cylinder

A Cross - Section is the intersection of a three-dimensional figure and a plane.

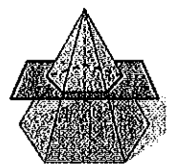
Describe the cross section.

Example 8:



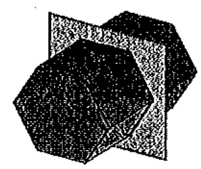
a point

Example 9:



pentagon

Example 10:



hexagon

**Quick Write: Write the definition of a cross section in your own words.*

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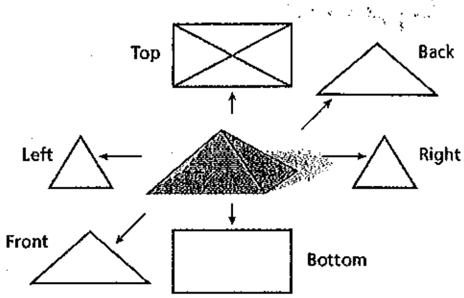
Homework Worksheet

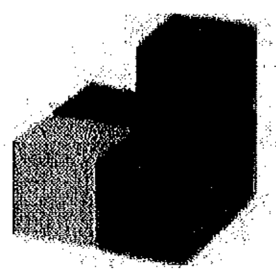
10.2 Representations of Three-Dimensional Figures

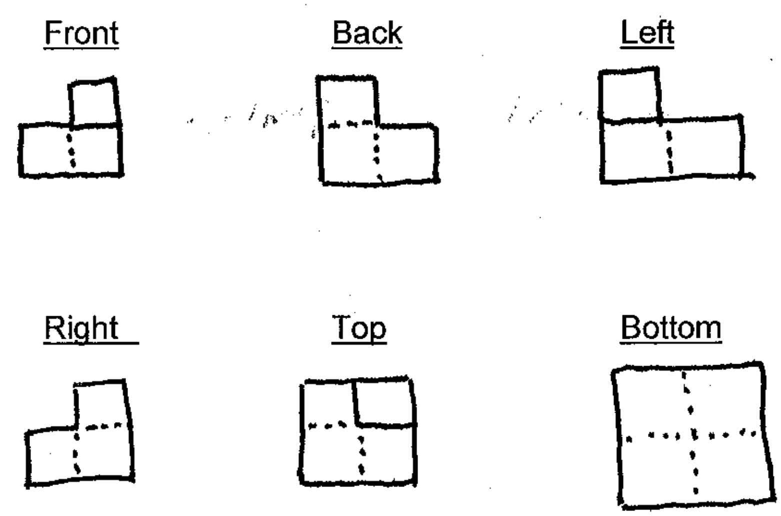
Learning Objectives: Students will draw representations of three-dimensional figures and recognize a three dimensional figure from a given representation.

There are many ways to represent a three dimensional object.
An orthographic drawing shows six different views of an object: top, bottom, front, back, left side, and right side.



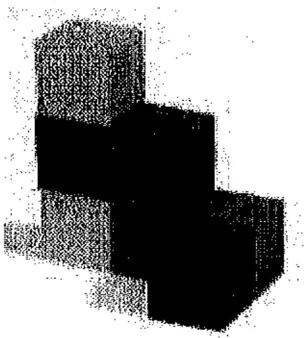
Example 1: Draw all six orthographic views of the given object. Assume there are no hidden cubes.

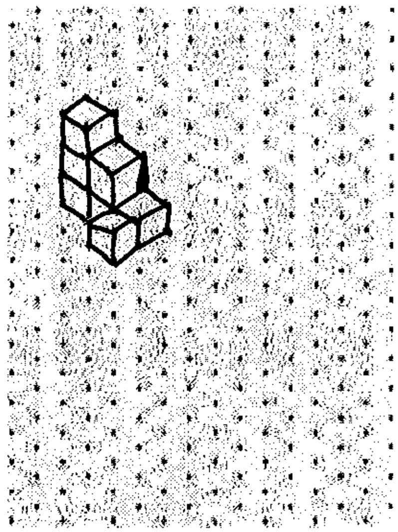




Isometric Drawing is a way to show three sides of a figure from a corner view. You can use *isometric dot paper* to make an isometric drawing. This paper has diagonal rows of dots that are equally spaced in a repeating triangular pattern.

Example 2: Draw an isometric view of the given object. Assume there are no hidden cubes.





**Quick Write: Isometric/Orthographic views, which is better and why?*

Homework: worksheet

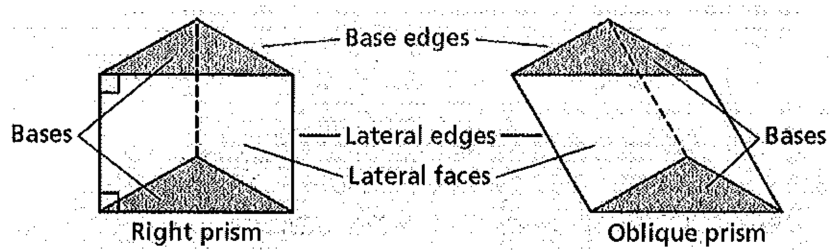
10.4 Surface Area of Prisms and Cylinders

Learning Objectives: Students will apply the formula for the surface area of a prism a cylinder.

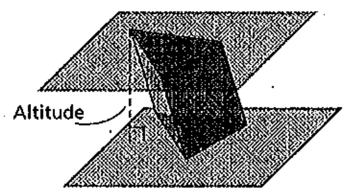
Prisms and Cylinders have 2 congruent parallel bases.

A *Lateral face* is not a base. The edges of the base are called base edges.

A *Lateral edge* is not an edge of a base, instead it *connects the two bases.* The lateral faces of a right prism are all ~~Reg~~ rectangles. An Oblique Prism has at least one nonrectangular lateral face.



An altitude of a prism or cylinder is a perpendicular segment joining the planes of the bases, this is the height of the figure.



The height MUST by perpendicular to the base (or to the plane that contains the base).

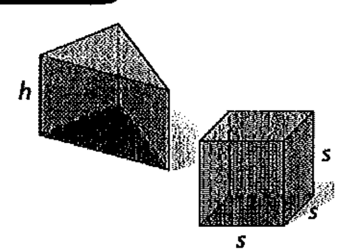
Surface Area is the *total area of all faces* and curved surfaces of a three-dimensional figure. The Lateral Areas of a prism is the sum of the areas of ONLY the *lateral faces.*

Lateral Area and Surface Area of Right Prisms

The lateral area of a right prism with base perimeter P and height h is $L = Ph$.

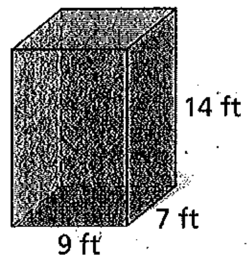
The surface area of a right prism with lateral area L and base area B is $S = L + 2B$, or $S = Ph + 2B$.

The surface area of a cube with edge length s is $S = 6s^2$.



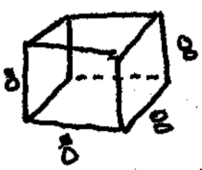
The surface area formula is only true for right prisms. To find the surface area of an oblique prism, add the areas of the faces.

Example 1: Find the lateral area and surface area of the right rectangular prism. Round to the nearest tenth, if necessary. YOU MUST SKETCH THE BASE.



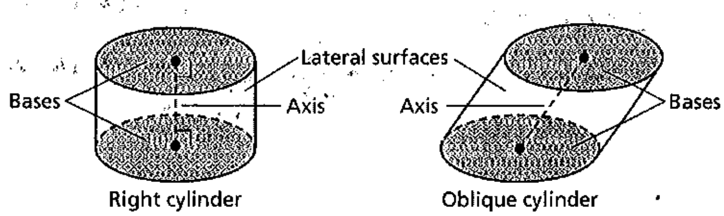
Lateral Area		Base (Sketch) - Area		(Total)Surface Area
$L = Ph$	+	$B = bh$	=	$S = Ph + 2B$ or
$P = 2(9) + 2(7) = 32$				$S = L + B(2)$
$h = 14$				
$L = 32(14) = 448\ ft^2$		$9(7) = 63$		$S = 448 + 2(63)$
		~~448 + 63 =~~		$S = 574\ ft^2$

Example 2: Draw a cube. Find the lateral area and surface area if the length of the edge is 8 cm. YOU MUST SKETCH THE BASE.



Lateral Area	Base (Sketch) -Area	(Total)Surface Area
$L = Ph$	 $B = bh$	$S = L + 2(B)$
$P = 2(8) + 2(8) = 32$	$= 8(8) = 64$	$= 256 + 2(64)$
$h = 8$		
$L = 32(8) = 256\ ft^2$		$S = 384\ ft^2$

The Lateral Surface of a cylinder is the curved surface that connects the two bases. The Axis of a Cylinder is the segment with endpoints at the centers of the bases. The axis of a right cylinder is perpendicular to its bases. The axis of an oblique Cylinder is not perpendicular to its bases. The altitude of a right cylinder is the same length as the axis.

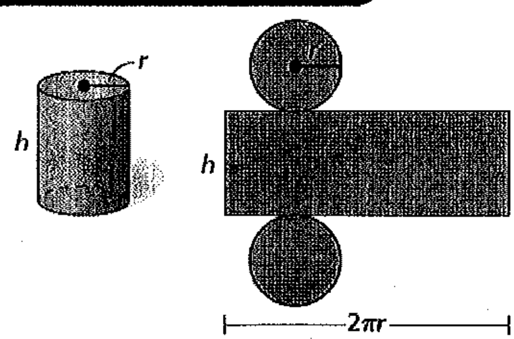


Right cylinder Oblique cylinder

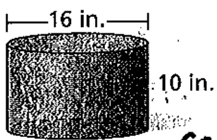
Lateral Area and Surface Area of Right Cylinders

The lateral area of a right cylinder with radius r and height h is $L = 2\pi rh$.

The surface area of a right cylinder with lateral area L and base area B is $S = L + 2B$, or $S = 2\pi rh + 2\pi r^2$.



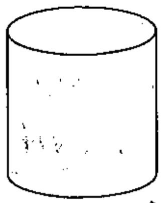
Example 3: Find the lateral area and surface area of the right cylinder. Give your answers in terms of π.



$r=8$
$h=10$

Lateral Area	Base (Sketch)-Area	(Total)Surface Area
$L=2\pi rh$	$B=\pi r^2$	$S=L+2\pi r^2$
$=2(8)(10)$	$=8^2\pi$	$S=160\pi+2\pi(8)^2$
$L=160\pi\ in^2$	$B=64\pi$	$S=288\pi\ in^2$

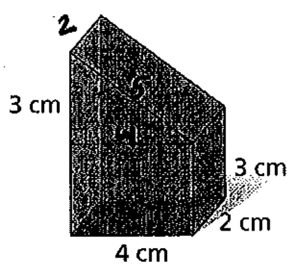
Example 4: Find the lateral area and surface area of a cylinder with a base area of 49π and a height that is 2 times the radius.(Label the sketch)



$\sqrt{49\pi}=\pi r^2$
$r=7$

Lateral Area	Base (Sketch)-Area	(Total)Surface Area
$L=2\pi rh$	$B=\pi r^2$	$S=L+2\pi r^2$
$=2(7)(14)$	$=(7^2)\pi$	$S=196\pi+2(49\pi)$
$L=196\pi$	$B=49\pi$	$S=294\pi$

Example 5: Find the surface area of the composite figure.



Rect. Prism	Trangular Prism
$S=Ph+2B$	$S=Ph+2B$
$P=2(4)+2(2)=12$	$P=3+4+5=12$
$h=3$	$h=2$
$B=4\cdot 2=8$	$B=\frac{1}{2}bh=\frac{1}{2}(3)(4)=6$
$S=12(3)+2(8)=52\ cm^2$	$S=12(2)+2(6)=36\ cm^2$
-8	-8
$44\ cm^2$	$28\ cm^2$

44
$+28$
$\boxed{72\ cm^2}$

**Quick Write: (Compare/Contrast) Discuss how prisms/cylinders are similar and how they are different.*

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Homework: Worksheet

10.5 Surface Area of Pyramids and Cones

Learning Objectives: Students will apply the formula for the surface area of a pyramid a cone.

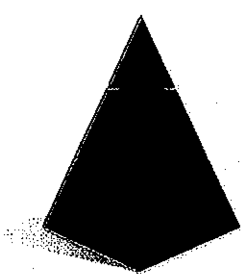
The vertex of a pyramid is the point (peak) opposite the base of the pyramid. The base of a *regular pyramid* is a regular polygon, and the lateral faces are *congruent isosceles triangles*. The slant height of a reg. pyramid is the distance from the vertex to the midpoint of an edge of the base. The altitude of a pyramid is the perpendicular segment from the vertex to the plane of the base.

Lateral and Surface Area of a Regular Pyramid

The lateral area of a regular pyramid with perimeter P and slant height ℓ is $L = \frac{1}{2}P\ell$.

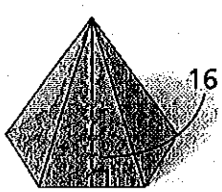
The surface area of a regular pyramid with lateral area L and base area B is $S = L + B$, or $S = \frac{1}{2}P\ell + B$.

Example 1: Find the lateral area and surface area of a regular square pyramid with base edge length 14 cm and slant height 25 cm. Round to the nearest tenth, if necessary.



Lateral Area	Base (Sketch)-Area	(Total)Surface Area
$L = \frac{1}{2}P\ell$	14, 14 $B = bh$	$S = L + B$
$P = 4(14)$	$= 14(14)$	$S = 700 + 196$
$\ell = 25$	$B = 196$	$S = 896\ cm^2$
$L = \frac{1}{2}(56)25 = 700\ cm^2$		

Example 2: Find the lateral area and surface area of the regular pyramid.

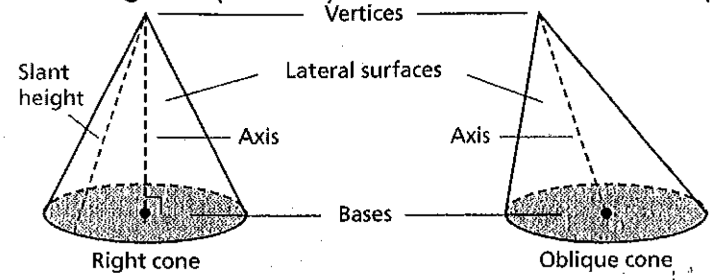


16 in.

10 in.

Lateral Area	Base (Sketch)-Area	(Total)Surface Area
$L = \frac{1}{2}P\ell$	$B = \frac{1}{2}aP$	$S = L + B$
$P = (10)6 = 60$	$\frac{1}{2}(5\sqrt{3})(60)$ 5√3, 5	$S = 480 + 259.81$
$\ell = 16$	$B = 259.81$	$S = 739.8\ in^2$
$L = \frac{1}{2}(60)16 = 480\ in^2$		

The vertex of a cone is the point (peak) opposite the base. The axis of a cone is the segment with endpoints at the vertex and the center of the base. The axis of a *Right Cone* is perpendicular to the base. The axis of an *Oblique Cone* is *not* perpendicular to the base.

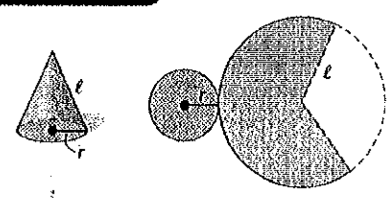


The Slant height of a right cone is the distance from the vertex of a right cone to a point on the edge of the base. The altitude of a cone is a perpendicular segment from the vertex of the cone to the plane of the base (the true height).

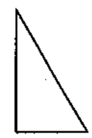
Lateral and Surface Area of a Right Cone

The lateral area of a right cone with radius r and slant height ℓ is $L = \pi r \ell$.

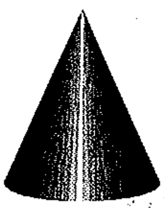
The surface area of a right cone with lateral area L and base area B is $S = L + B$, or $S = \pi r \ell + \pi r^2$.



Relationship between dimensions in a cone.



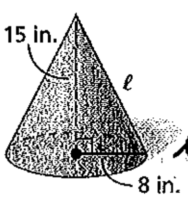
Example 3: Find the lateral area and surface area of a right cone with radius 9 cm and slant height 5 cm.



$r=9$
$\ell=5$

Lateral Area	Base (Sketch)-Area	(Total)Surface Area
$L=\pi r\ell$	$B=\pi r^2$	$S=L+B$
$L=\pi(9)(5)$	$B=9^2\pi$	$S=45+81$
$L=45\pi\ cm^2$	$B=81\pi$	$S=126\pi\ cm^2$

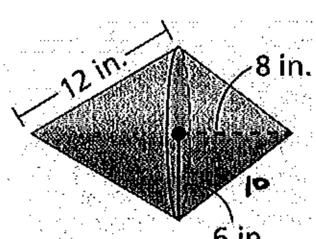
Example 4: Find the lateral area and surface area of the cone.



$\ell:17$
$r=8$
$8^2+15^2=\ell^2$

Lateral Area	Base (Sketch)-Area	(Total)Surface Area
$L=\pi r\ell$	$B=\pi r^2$	$S=L+B$
$=8(17)$	$=8^2\pi$	$S=136\pi+64\pi$
$L=136\pi\ in^2$	$B=64\pi$	$S=200\pi\ in^2$

Example 5: Finding Surface Area of Composite Three-Dimensional Figures



Left hand cone:	Right hand cone:	Total Area:
$S=L+B$	$S=L+B$	72π
$S=\pi r\ell+\pi r^2$	$S=\pi r\ell+\pi r^2$	$+60\pi$
$S=\pi 6(12)+6^2\pi$	$S=\pi(6)(10)+\pi 6^2$	
$S=72\pi$	$S=60\pi$	$132\pi\ in^2$

**Quick Write: What is the difference between slant height and "true" height? How do you find one given the other?*

Homework: Worksheet

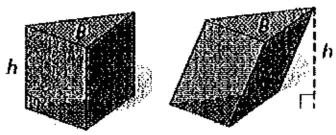
10.6 Volume of Prisms and Cylinders

Learning Objectives: Students will apply the formula for the volume of a prism and a cylinder.

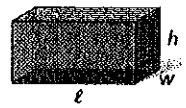
The Volume of a three-dimensional figure is the number of nonoverlapping unit cubes of a given size that will exactly fill the interior.

Volume of a Prism

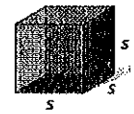
The volume of a prism with base area B and height h is $V = Bh$.



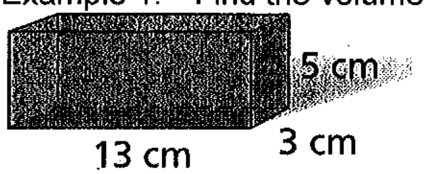
The volume of a right rectangular prism with length ℓ, width w, and height h is $V = \ell wh$.



The volume of a cube with edge length s is $V = s^3$.

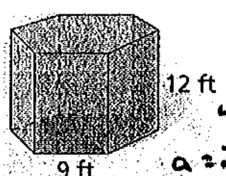


Example 1: Find the volume of the prism.



Base (Sketch)-Area	Height	Volume
3, 13: $3(13) = 39$	5	$V = \ell wh$ $V = 39(5)$ $V = 195 \text{ cm}^3$

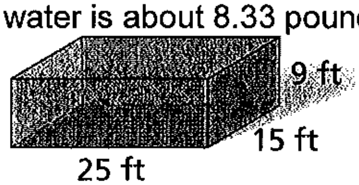
Example 2: Find the volume of the right regular hexagonal prism. Round to the nearest tenth.



$4.5\sqrt{3}$
$a = \cancel{3\sqrt{3}}$
$P = 54$

Base (Sketch)-Area	Height	Volume
9, $4.5\sqrt{3}$: $B = \frac{1}{2}(\cancel{3\sqrt{3}}\ 4.5\sqrt{3})(54)$ $= \cancel{140.3}$ $= 210.44$	12	$V = \cancel{140.3}\ 210.44(12)$ $V = \cancel{1683.6 \text{ ft}^3}$ 2525.3 ft^3

Example 3: A swimming pool is a rectangular prism. Estimate the volume of water in the pool in gallons when it is completely full (Hint: 1 gallon ≈ 0.134 ft3). The density of water is about 8.33 pounds per gallon. Estimate the weight of the water in pounds.

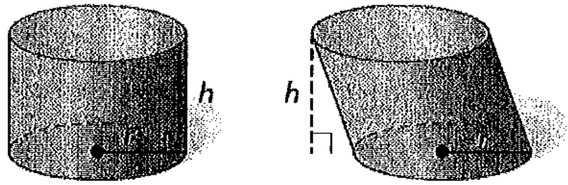


$V = 25(15)(9) = 3375 \text{ ft}^3 \cdot \frac{1}{0.134} \approx 25{,}187 \text{ gal.}$

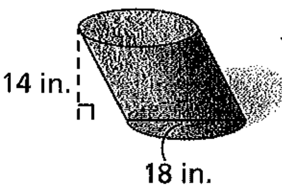
$25{,}187 \cdot \frac{8.33}{1} = 209{,}807$ pounds

Volume of a Cylinder

The volume of a cylinder with base area B, radius r, and height h is $V = Bh$, or $V = \pi r^2 h$.

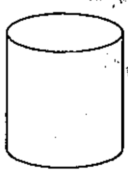


Example 4: Find the volume of the cylinder. Give your answers in terms of π and rounded to the nearest tenth.



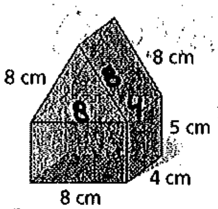
Base (Sketch)-Area	Height	Volume
9 $B=\pi r^2$ $=9^2\pi$ $B=81\pi$	14	$V=\pi r^2 h$ $V=81(14)\pi$ $V=1134\pi \text{ in}^3$ 3562.6 in^3

Example 5: Find the volume of a cylinder with a diameter of 16 in. and a height of 17 in. Give your answer both in terms of π and rounded to the nearest tenth.



Base (Sketch)-Area	Height	Volume
8 $B=\pi r^2$ $=8^2\pi$ $B=64\pi$	17	$V=64(17)\pi$ $V=1088\pi \text{ in}^3$ 3418.1 in^3

Example 6: Find the volume of the composite figure. Round to the nearest tenth.



Vol. of Tri. Prism	Vol. of Rec. Prism	Total Volume
$V=lwh$ $V=Bh$ $V=27.6(4)=110.4$	$V=lwh$ $V=8(4)(5)=160$	110.4 $+160$ 270.4 cm^3

$h=4$ $B=\frac{1}{2}bh$ $\frac{1}{2}(8)6.9=27.6$ ~~32+32=64~~ ~~subtract~~

$16+h^2=64$

Quick Write: Give a real world example of when you would need to find area and volume.

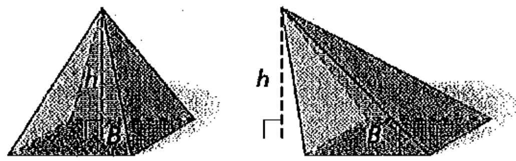
Homework: Worksheet

10.7 Volume of Pyramids and Cones

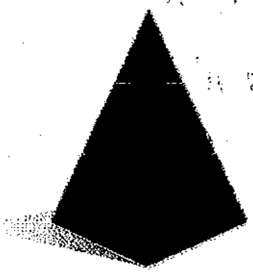
Learning Objectives: Students will apply the formula for the volume of a pyramid and a cone.

Volume of a Pyramid

The volume of a pyramid with base area B and height h is $V = \frac{1}{3}Bh$.

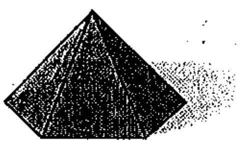


Example 1: Find the volume a rectangular pyramid with length 11 m, width 18 m, and height 23 m.

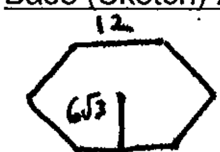


Base (Sketch)-Area	Height	Volume
$B = lw$	23	$V = \frac{1}{3}Bh$
$B = 198$		$V = \frac{1}{3}(198)23$
		$V = 1518\ m^3$

Example 2: Find the volume of the regular hexagonal pyramid with height equal to the apothem of the base.

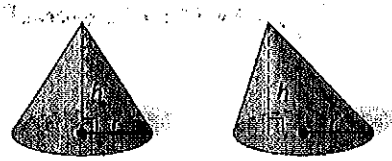


12 ft

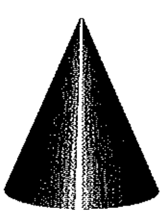
Base (Sketch)-Area	Height	Volume
	$6\sqrt{3}$	$V = \frac{1}{3}Bh$
		$V = \frac{1}{3}(374.12)(6\sqrt{3})$
$B = \frac{1}{2}(6\sqrt{3})(72) = 374.12$		$V = 1296\ ft^3$

Volume of Cones

The volume of a cone with base area B, radius r, and height h is $V = \frac{1}{3}Bh$, or $V = \frac{1}{3}\pi r^2 h$.

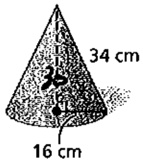


Example 3: Find the volume of a cone with radius 7 cm and height 15 cm. Give your answers both in terms of π and rounded to the nearest tenth.



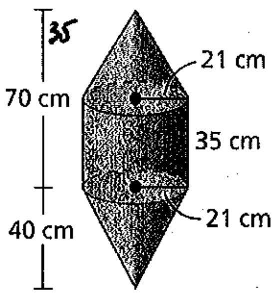
Base (Sketch)-Area	Height	Volume
7 $r = 7$	15	$V = \frac{1}{3}Bh$
$B = \pi r^2$		$V = \frac{1}{3}(49)(15)$
$B = 49\pi$		$V = 245\pi\ cm^3$
		$769.7\ \pi$

Example 4: Find the volume of a cone.



Base (Sketch)-Area	Height	Volume
$B=\pi r^2$	30	$V=\frac{1}{3}Bh$
$B=16^2\pi$		$V=\frac{1}{3}(256)(30)$
$B=256\pi$		$V=2560\pi\ cm^3$
		$8042.47\ cm^3$

Example 5: Find the volume of the composite figure. Round to the nearest tenth.



Vol. cone1	vol. of cyl.	Vol. of cone 2	Total Volume
$V=\frac{1}{3}Bh$	$V=\pi r^2 h$	$V=\frac{1}{3}Bh$	5145
$V=\frac{1}{3}(21^2)35$	$V=(21^2)35$	$V=\frac{1}{3}(21^2)(40)$	15435
$V=5145\pi$	$V=15435\pi$	$V=5888\pi$	+ 5888
			$=26468\pi\ cm^3$
			$83151.6\ cm^3$

**Quick Write:* *How much greater is the volume of a cylinder than that of a cone with the same radius and height? Why?*

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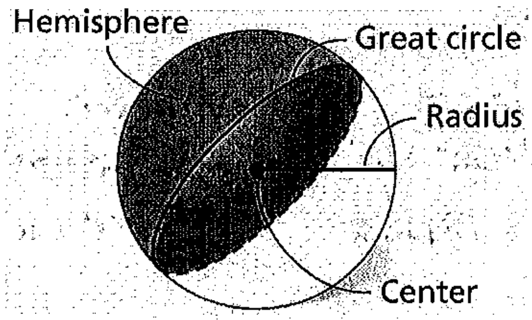
Homework: Worksheet

10.8 Spheres

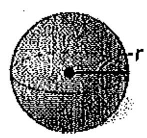
Learning Objectives: Students will apply the formula for the volume and surface area of a sphere.

A Sphere is the locus of points in space that are a fixed distance from a given point called the Center of a sphere. A radius of a sphere connects the center of the sphere to any point on the sphere. A hemisphere is half of a sphere.
A great circle divides a sphere into two hemispheres.

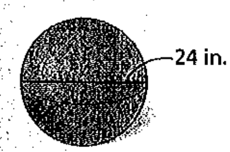


Volume of a Sphere

The volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.

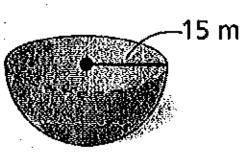


Example 1: Find the volume of the sphere. Give your answer in terms of π.



Radius	Volume of the Sphere
$r = 12$	$V = \frac{4}{3}\pi r^3$ $V = \frac{4}{3}(12^3)$ $V = 2304\pi \text{ in}^3$

Example 2: Find the volume of the hemisphere.



Radius	Volume of the Hemi-Sphere
$r = 15$	$V = \frac{4}{3}\pi r^3$ $V = \frac{4}{3}(15^3)$

$V = \frac{4500\pi}{2} \text{ m}^3 = 2250\pi \text{ m}^3$

Example 3: Find the radius of a sphere with volume $2304\pi \text{ ft}^3$.

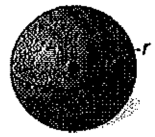
$$\frac{2304\pi}{4/3} = \frac{\frac{4}{3}\pi r^3}{4/3}$$

$$\sqrt[3]{1728} = \sqrt[3]{r^3}$$

$r = 12$

Surface Area of a Sphere

The surface area of a sphere with radius r is $S = 4\pi r^2$.



Example 4: Find the surface area of a sphere with diameter 76 cm. Give your answers in terms of π.

Radius	Surface Area
$76 \div 2 = 38$	$S = 4\pi r^2$ $S = 4(38^2)$ $S = 5776\pi \text{ cm}^2$

Example 5: Find the volume of a sphere with surface area 324π in^2. Give your answers in terms of π.

$\frac{324\cancel{\pi}}{4} = \frac{4\cancel{\pi} r^2}{4}$

$\sqrt{81} = \sqrt{r^2}$

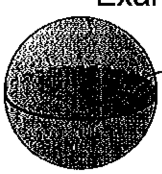
$r = 9$

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}(9^3)\pi$

$V = 972\pi \text{ in}^3$

Example 6: Find the surface area of a sphere with a great circle that has an area of 49π mi^2.



$\sqrt{\cancel{\pi} r^2} = \sqrt{49\cancel{\pi}}$

$r = 7$

$S = 4(7^2)\pi$

$S = 196\pi \text{ mi}^2$

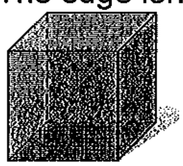
**Quick Write: In what way(s) do a circle and a sphere differ?*

Homework Worksheet

Changing Dimensions Proportionally

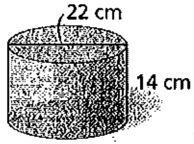
Change in Dimensions	Perimeter or Circumference	Area	Volume
All dimensions multiplied by a	Changes in factor of a	Changes in factor of a^2	Changes in factor of a^3

Example 1: The edge length of the cube is tripled. Describe the effect on the surface area.



surface area is multiplied by 3^2 or 9

Example 2: The height and diameter of the cylinder are multiplied by 1/2. Describe the effect on the surface area.

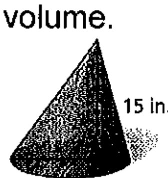


multiplied by $\left(\frac{1}{2}\right)^2$

Example 3: The length, width, and height of the prism are doubled. Describe the effect on the volume.

$(2)^3$ or multiplied by 8

Example 4: The diameter and height of the cone are divided by 3. Describe the effect on the volume.



divided by 3^3 or 27