3.1 Lines and Angles

Learning Goal: Identify parallel, perpendicular, and skew lines and identify angles formed by two lines and a transversal.

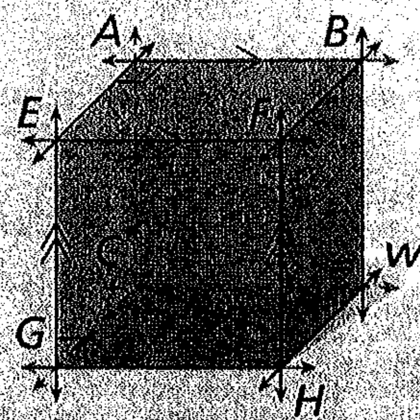
Parallel, Perpendicular, and Skew Lines

Parallel lines (||) are coplanar and do not intersect. In the figure, $\overleftrightarrow{AB} \parallel \overleftrightarrow{EF}$, and $\overleftrightarrow{EG} \parallel \overleftrightarrow{FH}$.

Perpendicular lines (⊥) intersect at 90° angles. In the figure, $\overleftrightarrow{AB} \perp \overleftrightarrow{AE}$, and $\overleftrightarrow{EG} \perp \overleftrightarrow{GH}$.

Skew lines are not coplanar. Skew lines are not parallel and do not intersect. In the figure, $\overleftrightarrow{AB}$ and $\overleftrightarrow{EG}$ are skew.

Parallel planes are planes that do not intersect. In the figure, plane *ABE* || plane *CDG*.



Arrows are used to show that $\overleftrightarrow{AB} \parallel \overleftrightarrow{EF}$ and $\overleftrightarrow{EG} \parallel \overleftrightarrow{FH}$.

Example #1: Identify the following:

A. A pair of Parallel Segments

$\overline{KN} \parallel \overline{PS}$

B. A pair of skew segments

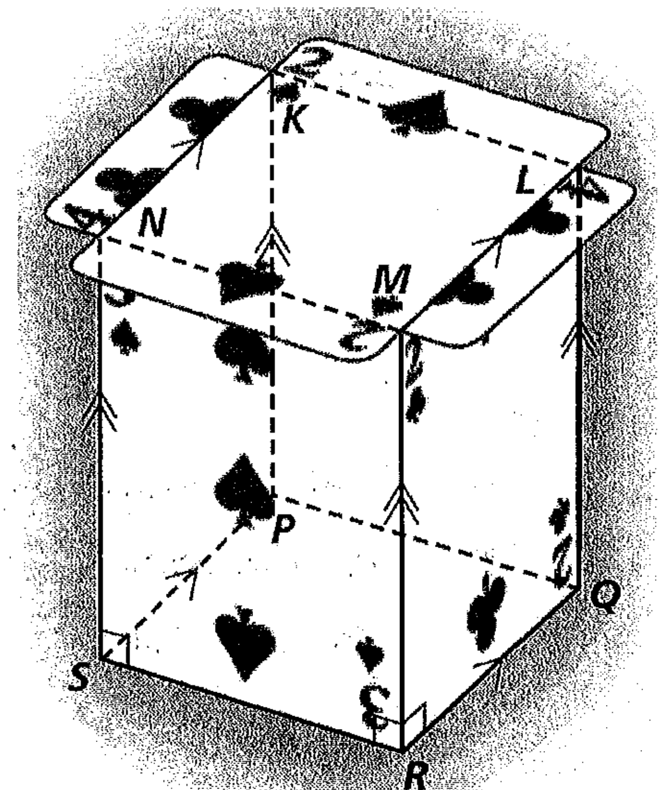
$\overline{LM}$ and $\overline{RS}$ are Skew

C. A pair of perpendicular segments

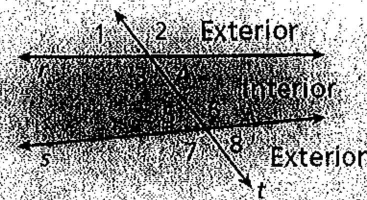
$\overline{MR} \perp \overline{RS}$

D. A pair of parallel planes

plane KPS || plane LQR



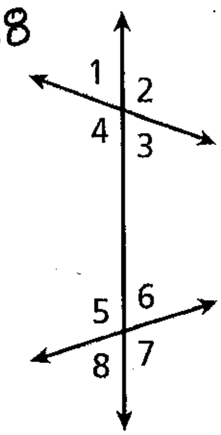
Angle Pairs Formed by a Transversal

TERM	EXAMPLE
A **transversal** is a line that intersects two coplanar lines at two different points. The transversal *t* and the other two lines *r* and *s* form eight angles.	
Corresponding angles lie on the same side of the transversal *t*, on the same sides of lines *r* and *s*.	∠1 and ∠5
Alternate interior angles are nonadjacent angles that lie on opposite sides of the transversal *t*, between lines *r* and *s*.	∠3 and ∠6
Alternate exterior angles lie on opposite sides of the transversal *t*, outside lines *r* and *s*.	∠1 and ∠8
Same-side interior angles or *consecutive interior angles* lie on the same side of the transversal *t*, between lines *r* and *s*.	∠3 and ∠5

Example #2: Give an example of each angle pair.

A. Corresponding angle ∠4 + ∠8

B. Alternate interior angles ∠4 and ∠6



C. Alternate exterior angles ∠2 + ∠8

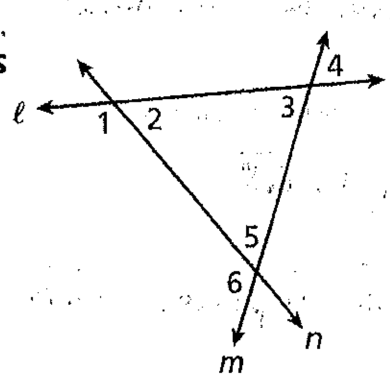
D. Same-side interior angles ∠4 and ∠5

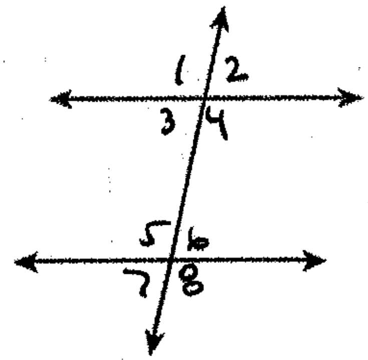
Example #3: Identify the transversal and classify each angle pair

A. ∠1 and ∠3 ℓ Corresponding ∠s

B. ∠2 and ∠6 n alt. interior ∠s

C. ∠4 and ∠6 m alt. exterior ∠s





∠ Pairs

Angle Pairs

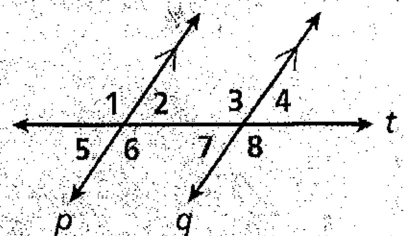
CA	AIA	AEA	SSIA
Corresponding ∠s	Alt. Int. ∠s	Alt. Ext ∠s	SS int. ∠s
∠1 + ∠5	∠3 + ∠6	∠2 + ∠7	∠4 + ∠6
∠3 + ∠7	∠4 + ∠5	∠1 + ∠8	∠3 + ∠5
∠2 + ∠6			
∠4 + ∠8			

HW: P 149: 14-28 Even, 44-48

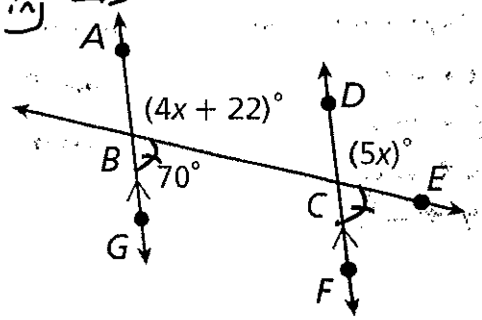
3.2 Angles Formed by Parallel Lines and Transversals

Learning Goal: Prove and use theorems about the angles formed by parallel lines and a transversal.

Postulate 3-2-1 Corresponding Angles Postulate

THEOREM	HYPOTHESIS	CONCLUSION
If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.		$\angle 1 \cong \angle 3$ $\angle 2 \cong \angle 4$ $\angle 5 \cong \angle 7$ $\angle 6 \cong \angle 8$

Example #1: Find Each angle Measure



A. $m\angle ECF$ $\angle ECF = \angle CBG$ Corresponding $\angle$'s

$m\angle ECF = 70°$

B. $m\angle DCE$ $= m\angle ABC$ Corresponding $\angle$s

$5x = 4x + 22$

$-4x \quad -4x$

$x = 22$

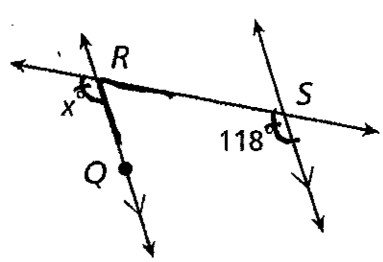
$m\angle DCE = 5x$

$= 5(22)$

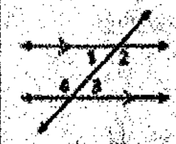
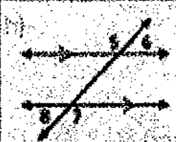
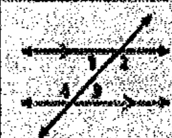
$m\angle DCE = 110°$

C. $m\angle QRS = 62°$ $x = 118$

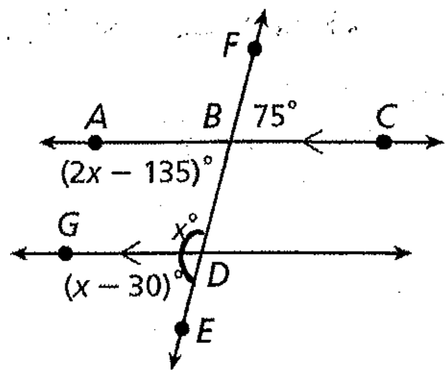
$180 - 118 = 62°$



Theorems Parallel Lines and Angle Pairs

THEOREM	HYPOTHESIS	CONCLUSION
3-2-2 Alternate Interior Angles Theorem If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.		$\angle 1 \cong \angle 3$ $\angle 2 \cong \angle 4$
3-2-3 Alternate Exterior Angles Theorem If two parallel lines are cut by a transversal, then the two pairs of alternate exterior angles are congruent.		$\angle 5 \cong \angle 7$ $\angle 6 \cong \angle 8$
3-2-4 Same-Side Interior Angles Theorem If two parallel lines are cut by a transversal, then the two pairs of same-side interior angles are supplementary.		$m\angle 1 + m\angle 4 = 180°$ $m\angle 2 + m\angle 3 = 180°$

Example #2: Find Each Measure:



A. $m\angle EDG$ = m∠DBA corresponding

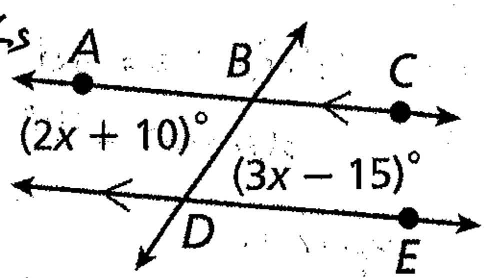
$2x - 135 = x - 30$
$-x \quad -x$
$x - 135 = -30$
$+135 \quad +135$
$x = 105$

$m\angle EDG = 105 - 30$

$m\angle EDG = 75°$

B. $m\angle BDG$

$180 - 75 = 105°$



C. $m\angle ABD$ + m∠EDB = 180 Alt. Int. ∠s

$2x + 10 + 3x - 15 = 180$

$5x - 5 = 180$
$+5 \quad +5$

$\frac{5x}{5} = \frac{185}{5}$

$x = 37$

$m\angle ABD = 2(37) + 10$

$m\angle ABD = 84°$

HW: P158 6-22 Even, 28, 32

3.3 Proving Lines Parallel

Learning Goal: Use angles formed by a transversal to prove two lines parallel.

With a partner write 2 conditional statements and their converse.

1. If two coplanar lines are cut by a transversal so that a pair of corr ≅∠s form, then the two lines are //.

Converse: ______________________

2. ______________________

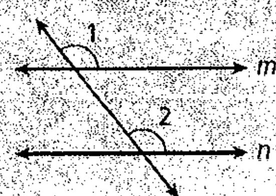
Converse: ______________________

Corresponding Angles Postulate:

If two parallel lines are cut by a transversal, then corresponding angles are congruent.

Converse: If corr. ∠s are ≅, then two // lines are cut by a transversal

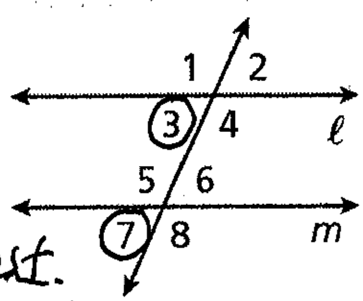
Postulate 3-3-1 **Converse of the Corresponding Angles Postulate**

THEOREM	HYPOTHESIS	CONCLUSION
If two coplanar lines are cut by a transversal so that a pair of corresponding angles are congruent, then the two lines are parallel.	$\angle 1 \cong \angle 2$ 	$m \parallel n$

Example #1: Use the Converse of the Corresponding Angles Postulate and the given information to show that ℓ || m.

m⍰3 = (4x – 80)°, 4(30) − 80 = 40°

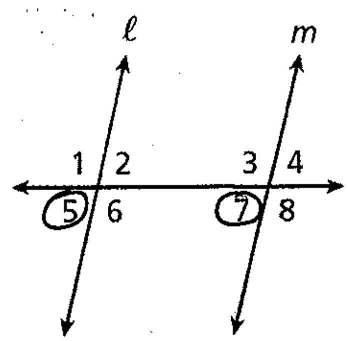
m⍰7 = (3x – 50)°, x = 30 3(30) − 50 = 40°



ℓ // m by Conv. of corr ∠s post.

m⍰7 = (4x + 25)°, 4(13) + 25 = 77°

m⍰5 = (5x + 12)°, x = 13 5(13) + 12 = 77°



ℓ // m by Conv. of Corr. ∠s post

With a partner, discuss when it is appropriate to use the Corresponding Angles Postulate, and when it is appropriate to use the converse. What is the difference between the two?

____

____

____

____

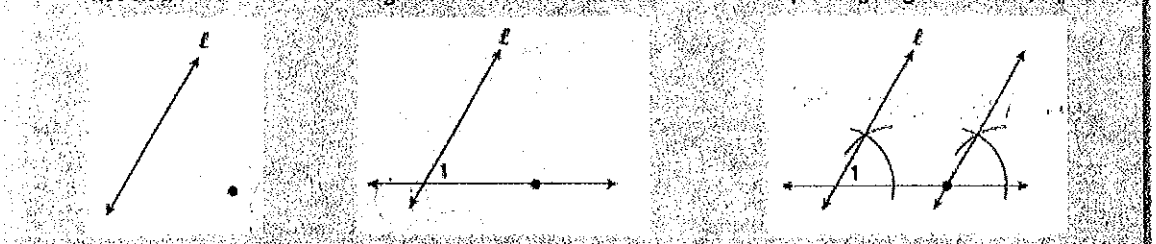
____

Postulate 3-3-2 Parallel Postulate

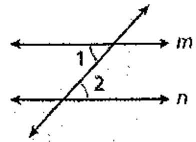
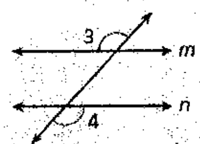
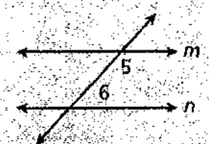
Through a point P not on line ℓ, there is exactly one line parallel to ℓ.

Construction Parallel Lines

1. Draw a line ℓ and a point P that is not on ℓ.
2. Draw a line m through P that intersects ℓ. Label the angle 1.
3. Construct an angle congruent to $\angle 1$ at P. By the converse of the Corresponding Angles Postulate, $\ell \parallel n$.



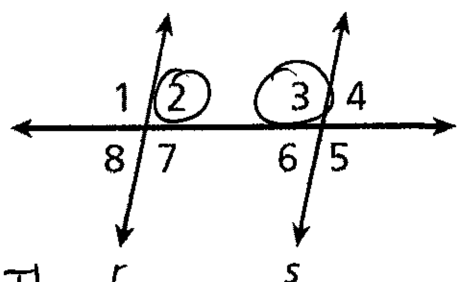
Theorems Proving Lines Parallel

	THEOREM	HYPOTHESIS	CONCLUSION
3-3-3	**Converse of the Alternate Interior Angles Theorem** If two coplanar lines are cut by a transversal so that a pair of alternate interior angles are congruent, then the two lines are parallel.	$\angle 1 \cong \angle 2$ 	$m \parallel n$
3-3-4	**Converse of the Alternate Exterior Angles Theorem** If two coplanar lines are cut by a transversal so that a pair of alternate exterior angles are congruent, then the two lines are parallel.	$\angle 3 \cong \angle 4$ 	$m \parallel n$
3-3-5	**Converse of the Same-Side Interior Angles Theorem** If two coplanar lines are cut by a transversal so that a pair of same-side interior angles are supplementary, then the two lines are parallel.	$m\angle 5 + m\angle 6 = 180°$ 	$m \parallel n$

Example #2: Use the given information and the theorems you have learned to show that r || s.

$m\angle 2 = (10x + 8)°$, Conv. SS Int. ∠s

$m\angle 3 = (25x - 3)°$, $x = 5$

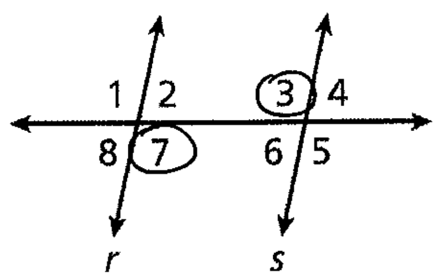


$10(5)+8+25(5)-3=180$

$180 = 180$

r // s Conv. Same Side Int. ∠s Thm.

$m\angle 3 = 2x°$, $m\angle 7 = (x + 50)°$, $x = 50$ Conv. Alt. Int. ∠s

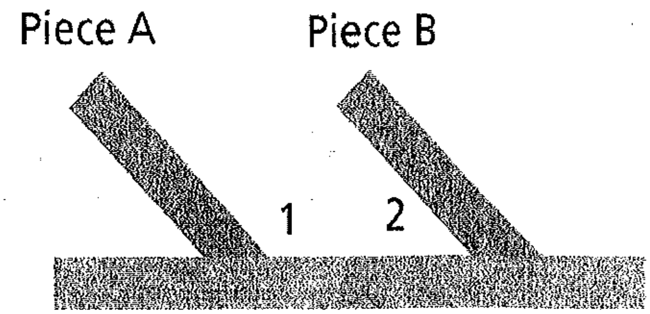


$2(50) = 50+50$

$100 = 100$

r // s Conv. Alt. Int. ∠s Thm.

Example #3:



A carpenter is creating a woodwork pattern and wants two long pieces to be parallel. m∠1= (8x + 20)° and m∠2 = (2x + 10)°. If x = 15, show that pieces A and B are parallel. Same Side Int. ∠s

$8(15)+20+2(15)+10=180$

$180=180$

piece A // piece B Conv. Same Side Int. ∠s Thm.

What are some angle pairs that must be congruent for the lines to be parallel?

Conv. of: Alt. Int. ∠s, Alt. Ext. ∠s, Corresponding ∠s

What are some angle pairs that must be supplementary for the lines to be parallel?

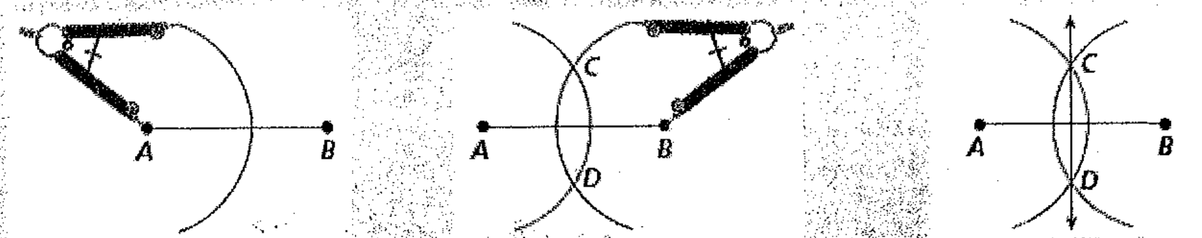
Conv. Same Side Int. ∠s.

HW: P167: 24-35, 42-45

3.4 Perpendicular Lines

Learning Goal: Apply Theorems about perpendicular lines

Construction Perpendicular Bisector of a Segment

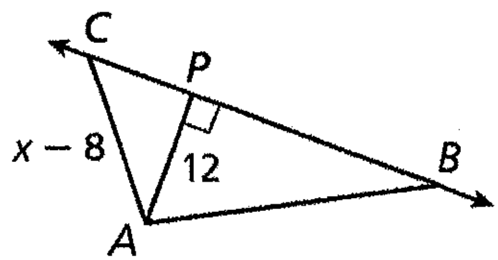


1. Draw $\overline{AB}$. Open the compass wider than half of AB and draw an arc centered at A.
2. Using the same compass setting, draw an arc centered at B that intersects the first arc at C and D.
3. Draw $\overleftrightarrow{CD}$. $\overleftrightarrow{CD}$ is the perpendicular bisector of $\overline{AB}$.

Example #1:

Name the shortest segment from point A to BC.

$\overline{AP}$



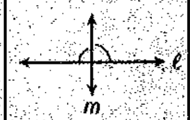
Write and solve an inequality for x.

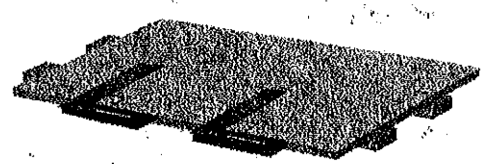
$AC > AP$

$x-8 > 12$

$x > 20$

Theorems

	THEOREM	GRAM	AMPLE
3-4-1	If two intersecting lines form a linear pair of congruent angles, then the lines are perpendicular. (2 intersecting lines form lin. pair of ≅ ∡ → lines ⊥.)		$\ell \perp m$
3-4-2	**Perpendicular Transversal Theorem** In a plane, if a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other line.		$q \perp p$
3-4-3	If two coplanar lines are perpendicular to the same line, then the two lines are parallel to each other. (2 lines ⊥ to same line → 2 lines ‖.)		$r \parallel s$



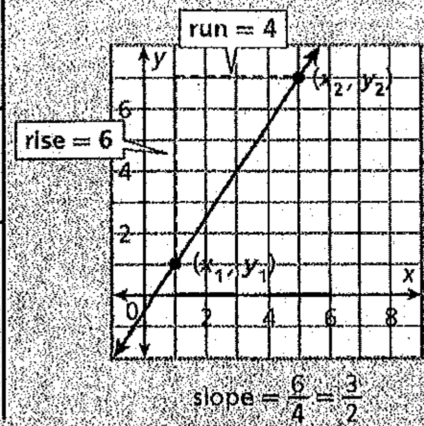
A carpenter's square forms a right angle. A carpenter places the square so that one side is parallel to an edge of a board, and then draws a line along the other side of the square. Then he slides the square to the right and draws a second line. Why must the two lines be parallel? Both lines are ⊥ to the edge of the board. If two coplanar lines are ⊥ to the same line, then the two lines are // to each other, so the lines must be // to each other.

HW: P175 6,10,21,29,30

3.5 Slopes of Lines

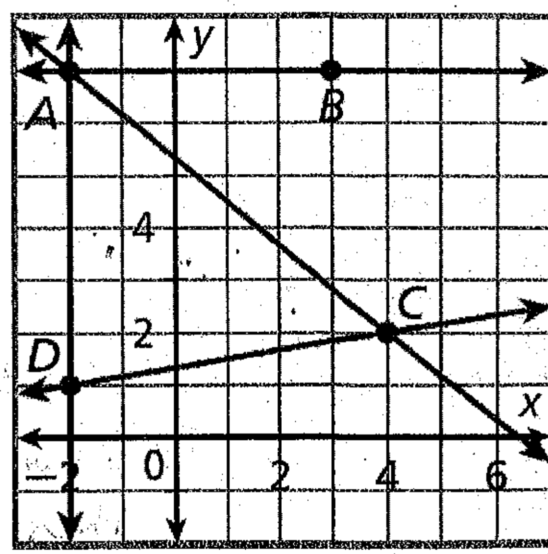
Learning Goal: Find slope and use it to identify parallel and perpendicular lines.

Slope of a Line

DEFINITION	EXAMPLE
The **rise** is the difference in the y-values of two points on a line.	
The **run** is the difference in the x-values of two points on a line.	
The **slope** of a line is the ratio of the rise to run. If (x_1, y_1) and (x_2, y_2) are any two points on a line, the slope of the line is $m = \frac{y_2 - y_1}{x_2 - x_1}$.	slope $= \frac{6}{4} = \frac{3}{2}$

$\frac{Y_2 - Y_1}{X_2 - X_1}$

Example #1: **Use the slope formula to determine the slope of each line.**



A. $\overleftrightarrow{AB}$ x y   A(−2, 7)   B(3, 7) $\frac{7-7}{3--2} = 0$

B. $\overleftrightarrow{AC}$ A(−2, 7)   C(4, 2) $\frac{2-7}{4--2} = -\frac{5}{6}$

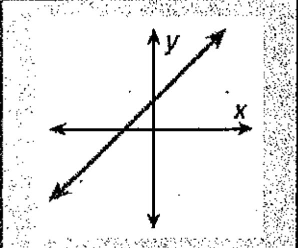
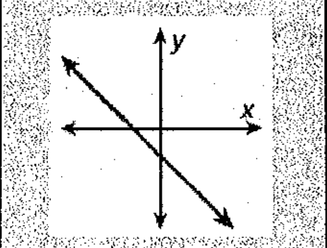
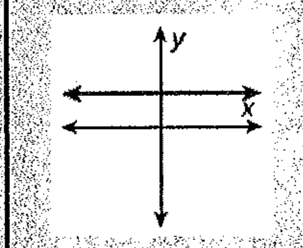
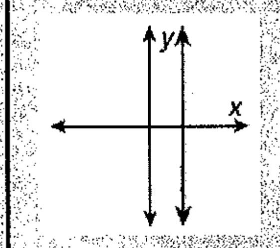
C. $\overleftrightarrow{AD}$ A(−2, 7)   D(−2, 1) $\frac{1-7}{-2--2} = \frac{-6}{0} =$ undefined

D. $\overleftrightarrow{CD}$ C(4, 2)   D(−2, 1) $\frac{1-2}{-2-4} = \frac{1}{6}$

Example #2: **Use the slope formula to determine the slope of *JK* through *J*(3, 1) and *K*(2, −1).**

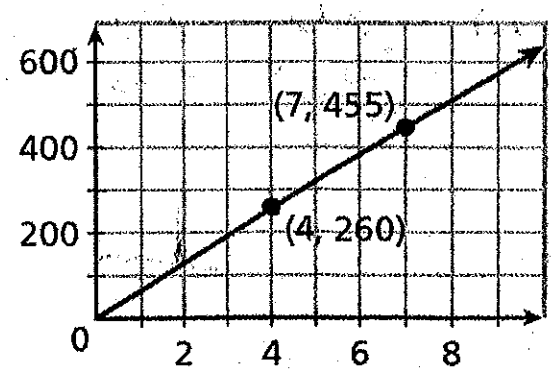
(3,1)
(2,−1)

$$\frac{y_2-y_1}{x_2-x_1} \qquad \frac{-1-1}{2-3} = 2$$

Summary: Slope of a Line			
Positive Slope	**Negative Slope**	**Zero Slope**	**Undefined Slope**
			

Example #3: **Justin is driving from home to his college dormitory. At 4:00 p.m., he is 260 miles from home. At 7:00 p.m., he is 455 miles from home. Graph the line that represents Justin's distance from home at a given time. Find and interpret the slope of the line.**

$$\frac{260-455}{4-7} = 65 \text{ mi/h}$$



Slopes of Parallel and Perpendicular Lines

3-5-1 Parallel Lines Theorem
In a coordinate plane, two nonvertical lines are parallel if and only if they have the same slope. Any two vertical lines are parallel.

3-5-2 Perpendicular Lines Theorem
In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is −1. Vertical and horizontal lines are perpendicular.

Example #4

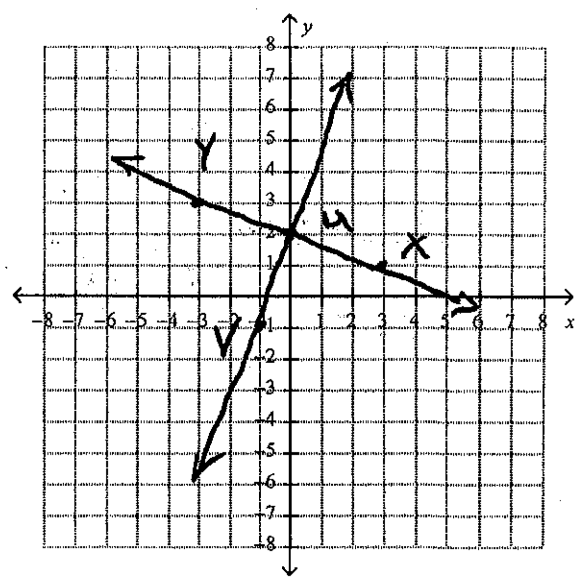
A. : **Graph each pair of lines. Use their slopes to determine whether they are parallel, perpendicular, or neither.**

$\overleftrightarrow{UV}$ **and** $\overleftrightarrow{XY}$ **for** ***U*(0, 2), *V*(−1, −1), *X*(3, 1), and *Y*(−3, 3)**

$\overleftrightarrow{UV}$ u(0,2) V(-1,-1) $\frac{-1-2}{-1-0} = 3$

$\overleftrightarrow{XY}$ (3,1) (-3,3) $\frac{3-1}{-3-3} = -\frac{1}{3}$

⊥



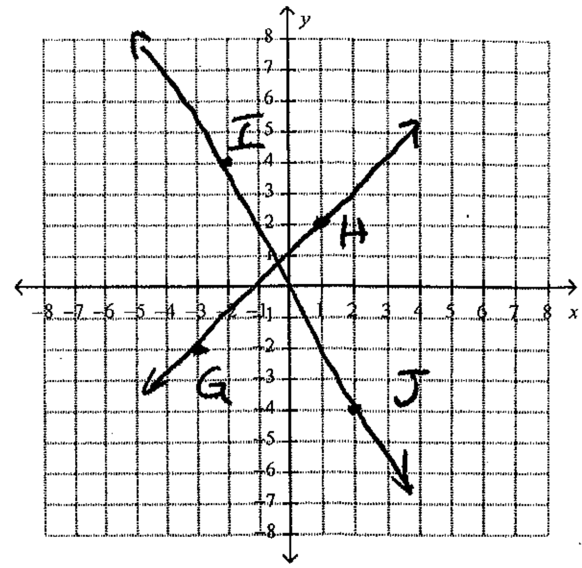
B. **Graph each pair of lines. Use their slopes to determine whether they are parallel, perpendicular, or neither.**

$\overleftrightarrow{GH}$ and $\overleftrightarrow{IJ}$ for $G(-3, -2)$, $H(1, 2)$, $I(-2, 4)$, and $J(2, -4)$

$\overleftrightarrow{GH}$ G(−3, −2) H(1, 2) $\frac{2 - {}^-2}{1 - {}^-3} = 1$

$\overleftrightarrow{IJ}$ I(−2, 4) J(2, −4) $\frac{{}^-4 - 4}{2 - {}^-2} = -2$

neither



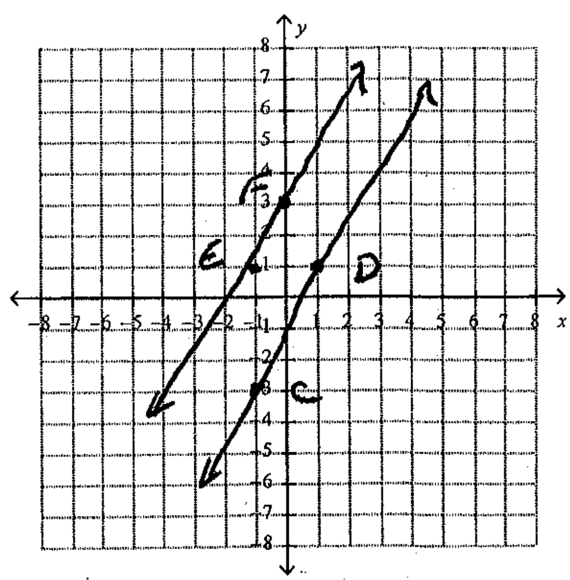
C. **Graph each pair of lines. Use their slopes to determine whether they are parallel, perpendicular, or neither.**

$\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ for $C(-1, -3)$, $D(1, 1)$, $E(-1, 1)$, and $F(0, 3)$

$\overleftrightarrow{CD}$ C(−1, −3) D(1, 1) $\frac{1 - {}^-3}{1 - {}^-1} = 2$

$\overleftrightarrow{EF}$ E(−1, 1) F(0, 3) $\frac{3 - 1}{0 - {}^-1} = 2$

parallel



Forms of the Equation of a Line

FORM	EXAMPLE
The **point-slope form** of a line is $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) is a given point on the line.	$y - 3 = 2(x - 4)$ $m = 2, (x_1, y_1) = (3, 4)$
The **slope-intercept form** of a line is $y = mx + b$, where m is the slope and b is the y-intercept.	$y = 3x + 6$ $m = 3, b = 6$
The equation of a vertical line is $x = a$, where a is the x-intercept.	$x = 5$
The equation of a horizontal line is $y = b$, where b is the y-intercept.	$y = 2$

Pairs of Lines

PARALLEL LINES	INTERSECTING LINES	COINCIDING LINES
$y = 5x + 8$ $y = 5x - 4$	$y = 2x - 5$ $y = 4x + 3$	$y = 2x - 4$ $y = 2x - 4$
Same slope different y-intercept	Different slopes	Same slope, same y-intercept

Example #5: Determine whether the lines are parallel, intersect, or coincide.

A. $y = 3x + 7$, $y = -3x - 4$

B. $y = -1/3x + 5$, $6y = -2x + 12$

C. $2y - 4x = 16$, $y - 10 = 2(x - 1)$

D. $3x + 5y = 2$, $3x + 6 = -5y$

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