2.1 Using Inductive Reasoning to Make Conjectures

Learning Goal: Use inductive reasoning to identify patterns, make conjectures and find counterexamples to disprove conjectures.

Video: http://player.discoveryeducation.com/index.cfm?guidAssetId=b1b5f95d-f72e-4d18-b220-ff73204e9a74

Term	Definition
Conjecture	a statement you believe to be true based on inductive reasoning.
Inductive Reasoning	Process of reasoning that a statement is true because specific cases are true.
Counter Example	Used to show a conjecture is false by finding one example.

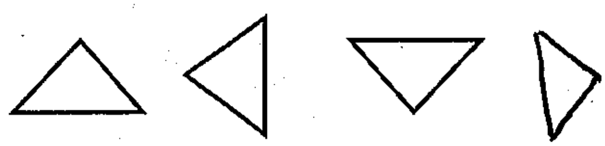
Example 1: A. Find the next item in the pattern.

January, March, May, ... July

B. Find the next item in the pattern

7,14,21,28,... 35

C. Find the next item in the pattern.



D: Find the next item in the pattern 0.4, 0.04, 0.004, ... 0.0004

Example 2: A. Complete the conjecture.

The sum of 2 positive numbers is ___?

positive

B. Complete the conjecture.

The number of lines formed by 4 points, no three of which are collinear, is __?

6

C. Complete the conjecture.

The product of two odd numbers is ___? odd

Example 3A: The cloud of water leaving a whale's blowhole when it exhales is called its *blow*. A biologist observed blue-whale blows of 25 ft, 29 ft, 27 ft, and 24 ft. Another biologist recorded humpback-whale blows of 8 ft, 7 ft, 8 ft, and 9 ft. Make a conjecture based on the data.

Heights of Whale Blows				
Height of Blue-whale Blows	25	29	27	24
Height of Humpback-whale Blows	8	7	8	9

The smallest blue-whale blow (24 ft) is almost three times higher than the greatest humpback-whale blow (9 ft).

Possible conjectures: The height of a blue whale's blow is greater than a humpback whale's.

Example 3B: Make a conjecture about the lengths of male and female whales based on the data.

Female whales are longer than male whales.

Average Whale Lengths						
Length of Female (ft)	49	51	50	48	51	47
Length of Male (ft)	47	45	44	46	48	48

To show that a conjecture is always true, you must prove it. To show that a conjecture is false, you have to find only one example in which the conjecture is not true. This case is called a counterexample. A counterexample can be a drawing, a statement, or a number.

Example 4: A. Show that the conjecture is false by finding a counterexample.

For every integer n, n^3 is positive. $n = -3$

B. Show that the conjecture is false by finding a counterexample.

Two complementary angles are not congruent. $45°$ and $45°$

C. Show that the conjecture is false by finding a counterexample. The monthly high temperature in Abilene is never below 90°F for two months in a row.

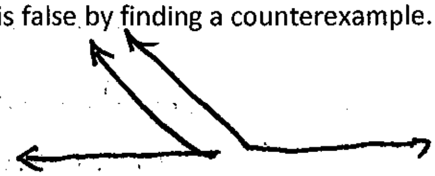
Jan. and Feb.

Monthly High Temperatures (°F) in Abilene, Texas											
Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
88	89	97	99	107	109	110	107	106	103	92	89

D. Show that the conjecture is false by finding a counterexample. For any real number x, $x^2 \geq x$.

$x = 1/2$

E. Show that the conjecture is false by finding a counterexample. Supplementary angles are adjacent.



F. Show that the conjecture is false by finding a counterexample. The radius of every planet in the solar system is less than 50,000 km.

Jupiter and Saturn

Planets' Diameters (km)								
Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto
4880	12,100	12,800	6790	143,000	121,000	51,100	49,500	2300

HW: P77, 2-24 Even

2.2/2.4 Conditional and Biconditional Statements

Learning Goal: Identify, write, and analyze the truth value of a conditional and biconditional statement.

Term and Definition	Draw it or Give Example
Conditional	hypothesis conclusion if p, then q
Hypothesis	part "p" of a conditional Statement
Conclusion	part "q" of a conditional Statement
Converse	Statement formed by exchanging the hypothesis and conclusion $q \to p$
Biconditional	Statement that can be written in the form: "p if and only if q" "if p, then q" "if q, then p"
Truth Value A conditional statement has a truth value of either true (T) or false (F). It is false only when the hypothesis is true and the conclusion is false.	
Counter Example To show that a conditional statement is false, you need to find only one counterexample where the hypothesis is true and the conclusion is false	

Conditional Statements

DEFINITION	SYMBOLS	VENN DIAGRAM
A **conditional statement** is a statement that can be written in the form "If *p*, then *q*."		
The **hypothesis** is the part *p* of a conditional statement following the word *If*.	$p \to q$	
The **conclusion** is the part *q* of a conditional statement following the word *then*.		

Example #1: ID the Hypothosis and aconclusion of each conditional

a. If today is Thanksgiving Dan, then today is Thursday.
p q

b. A number is a rational number if it is an integer.
q p

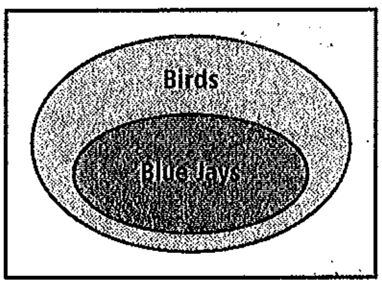
c. A number is divisible by 3 if it is divisible by 6.
q p

Example #2: Write a Conditional statement from the following

a. An obtuse triangle has exactly one obtuse angle

If a triangle is obtuse, then it has exactly one obtuse angle.

b.



If an animal is a blue jay, then it is a bird.

c. Two angles that are complementary are acute.

If 2 $\angle$'s are comp., then they are acute.

Example 3: Analyzing the Truth Value of a Conditional Statement. Determine if the conditional is true. If false, give a counterexample.

A. If this month is August, then next month is September. True

B. If an even number greater than 2 is prime, then $5 + 4 = 8$. True

C. It 2 angles are acute, then they are congruent/ False $80^\circ + 30^\circ$

D. If a number is odd, then it is divisible by 3. False 7

Example #4: Write the conditional statement and the converse within the biconditional

A. An angle is obtuse if and only if its measure is greater than 90° and less than 180°.

Cond: If an ∠ is obtuse, then its measure is greater than 90° and less than 180°.

Conv: If the measure of an ∠ is greater than 90° + less than 180°, then the ∠ is obtuse.

B. A solution is neutral ↔ its pH is 7.

Cond: If a solution is neutral, then its pH is 7.

Conv: If the pH of a solution is 7, then it is neutral.

C. An angle is acute iff its measure is greater than 0° and less than 90°.

Cond: If an ∠ is acute, then its measure is greater than 0° + less than 90°.

Conv: If an ∠'s measure is greater than 0° + less than 90°, then the ∠ is acute.

D. Cho is a member if and only if he has paid the $5 dues.

Cond: If Cho is a member, then he has paid the $5 dues.

Conv: If Cho has paid the $5 dues, then he is a member.

Example #5: For the conditional, write the converse and a biconditional statement

A. If points lie on the same line, then they are collinear.

Conv: If pts. are collinear, then they lie on the same line.

BiCond: Pts. lie on the same line if and only if they are collinear.

B. If two angles have the same measure, then they are congruent.

Conv: If 2 ∠s are ≅, then they have the same measure.

BiCond: 2 ∠s have the same measure if and only if they are ≅.

C. If $5x - 8 = 37$, then $x = 9$.

Conv: If $x=9$, then $5x-8=37$.

BiCond: $5x-8=37$ if and only if $x=9$.

D. If the date is July 4th, then it is Independence Day.

Conv: If it is Independence day, then the date is July 4th.

BiCond: It is July 4th if and only if it is Independence day.

Example #6: Wite your own conditional statement. Trade with a partner and have them write the converse.

Conditional: ______________________________

Converse: ______________________________

HW: P 84: 13-21, P99 6-9, 16-19

2.5 Algebraic Proof

Learning Goal: Review and Identify properties of Equality.

Properties of Equality

Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $ac = bc$.
Division Property of Equality	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Reflexive Property of Equality	$a = a$
Symmetric Property of Equality	If $a = b$, then $b = a$.
Transitive Property of Equality	If $a = b$ and $b = c$, then $a = c$.
Substitution Property of Equality	If $a = b$, then b can be substituted for a in any expression.

Remember!

The Distributive Property states that

$a(b + c) = ab + ac.$

Example #1:

A. **Solve the equation $4m - 8 = -12$. Write a justification for each step.**

$4m - 8 = -12$ Given

$+8 \quad +8$ Add Prop of =

$\frac{4m}{4} = \frac{-4}{4}$ Simplify

Div. Prop of =

$m = -1$ Simplify

B. **Solve the equation. Write a justification for each step.**

Example #2:

A. **What is the temperature in degrees Fahrenheit F when it is 15°C? Solve the equation $F = \frac{9}{5}C + 32$ for F and justify each step.**

$F = \frac{9}{5}C + 32$ Given

$F = \frac{9}{5}(15) + 32$ Substitution

$F = 27 + 32$ simplify

$F = 59$

B. **What is the temperature in degrees Celsius C when it is 86°F? Solve the equation $C = \frac{5}{9}(F - 32)$ for C and justify each step.**

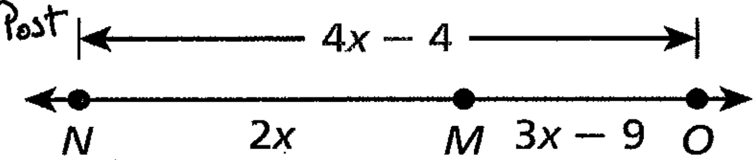
$C = \frac{5}{9}(F - 32)$ Given

$C = \frac{5}{9}(86 - 32)$ Subst.

$C = \frac{5}{9}(54)$ simplify

$C = 30$ simplify

C. **Write a justification for each step. Solve for X.**



$NO = NM + MO$ Seg. Add. Post

$4x - 4 = 2x + 3x - 9$ Subst.

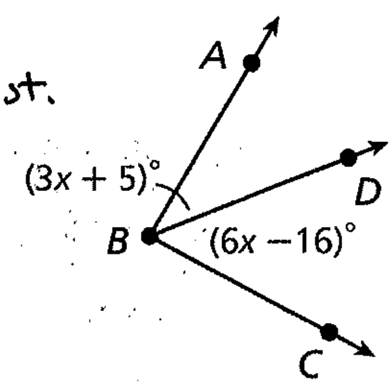
$4x - 4 = 5x - 9$ simplify

$-4 = x - 9$ Subtract Prop of =

$+9 \quad +9$ Add Prop of =

$5 = x$ simplify

D. **Write a justification for each step. Solve for X.**



$m\angle ABC = 8x°$

$m\angle ABC = m\angle ABD + m\angle DBC$ ∠ Add. Post.

$8x = (3x+5)° + (6x-16)°$ Subst.

$8x = 9x - 11$ Simplify

$-9x \quad -9x$ Subtracti Prop =

$\frac{-1x}{-1} = \frac{-11}{-1}$ Div. Prop of =

$x = 11$ Simplify

Remember!

Numbers are equal (=) and figures are congruent (≅).

Properties of Congruence

SYMBOLS	EXAMPLE
Reflexive Property of Congruence figure *A* ≅ figure *A* (Reflex. Prop. of ≅)	$\overline{EF} \cong \overline{EF}$
Symmetric Property of Congruence If figure *A* ≅ figure *B*, then figure *B* ≅ figure *A*. (Sym. Prop. of ≅)	If $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 1$.
Transitive Property of Congruence If figure *A* ≅ figure *B* and figure *B* ≅ figure *C*, then figure *A* ≅ figure *C*. (Trans. Prop. of ≅)	If $\overline{PQ} \cong \overline{RS}$ and $\overline{RS} \cong \overline{TU}$, then $\overline{PQ} \cong \overline{TU}$.

Quick Write: What is the difference between the Properties of Equality and the Properties of Congruence?

equality deals with number values and congruence deals with figures or shapes.

Identify the property that justifies each statement.

A. $\angle QRS \cong \angle QRS$ Reflex. Prop of ≅

B. $m\angle 1 = m\angle 2$ so $m\angle 2 = m\angle 1$ Sym. Prop of =

C. $AB \cong CD$ and $CD \cong EF$, so $AB \cong EF$. Trans. Prop. of ≅

D. $32° = 32°$ Reflex. Prop of =

E. $DE = GH$, so $GH = DE$. Sym. Prop of =

F. $94° = 94°$ Reflex. Prop of =

G. $0 = a$, and $a = x$. So $0 = x$. Trans. Prop of =

H. $\angle A \cong \angle Y$, so $\angle Y \cong \angle A$ Sym. Prop of ≅

HW: PG 108 16-24, 30-32

Learning Goal: Apply Theorems to solve problems.

A **theorem** is any statement that you can prove. Once you have proven a theorem, you can use it as a reason in later proofs.

Theorem

THEOREM	HYPOTHESIS	CONCLUSION
2-6-1 Linear Pair Theorem If two angles form a linear pair, then they are supplementary.	$\angle A$ and $\angle B$ form a linear pair.	$\angle A$ and $\angle B$ are supplementary.

Theorem

THEOREM	HYPOTHESIS	CONCLUSION
2-6-2 Congruent Supplements Theorem If two angles are supplementary to the same angle (or to two congruent angles), then the two angles are congruent.	$\angle 1$ and $\angle 2$ are supplementary. $\angle 2$ and $\angle 3$ are supplementary.	$\angle 1 \cong \angle 3$

Theorems

THEOREM	HYPOTHESIS	CONCLUSION
2-6-3 Right Angle Congruence Theorem All right angles are congruent.	$\angle A$ and $\angle B$ are right angles.	$\angle A \cong \angle B$
2-6-4 Congruent Complements Theorem If two angles are complementary to the same angle (or to two congruent angles), then the two angles are congruent.	$\angle 1$ and $\angle 2$ are complementary. $\angle 2$ and $\angle 3$ are complementary.	$\angle 1 \cong \angle 3$

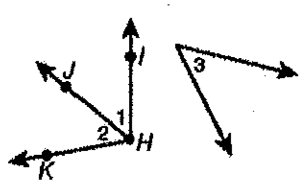
Theorem 2-7-1 Common Segments Theorem

THEOREM	HYPOTHESIS	CONCLUSION
Given collinear points A, B, C, and D arranged as shown, if $\overline{AB} \cong \overline{CD}$, then $\overline{AC} \cong \overline{BD}$. 	$\overline{AB} \cong \overline{CD}$	$\overline{AC} \cong \overline{BD}$

Theorems

THEOREM		HYPOTHESIS	CONCLUSION
2-7-2	**Vertical Angles Theorem** Vertical angles are congruent.	$\angle A$ and $\angle B$ are vertical angles.	$\angle A \cong \angle B$
2-7-3	If two congruent angles are supplementary, then each angle is a right angle. ($\cong$ ∡ supp. $\rightarrow$ rt. ∡)	$\angle 1 \cong \angle 2$ $\angle 1$ and $\angle 2$ are supplementary.	$\angle 1$ and $\angle 2$ are right angles.

Write the letter of the correct justification next to each step. (Use one justification twice.)

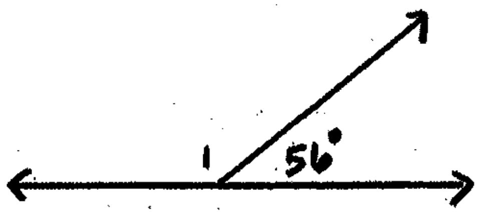


Given: $\overrightarrow{HJ}$ is the bisector of $\angle IHK$ and $\angle 1 \cong \angle 3$.

1. $\overrightarrow{HJ}$ is the bisector of $\angle IHK$. B.
2. $\angle 2 \cong \angle 1$ A.
3. $\angle 1 \cong \angle 3$ C.
4. $\angle 2 \cong \angle 3$ C.

A. Definition of $\angle$ bisector
B. Given
C. Transitive Prop. of $\cong$

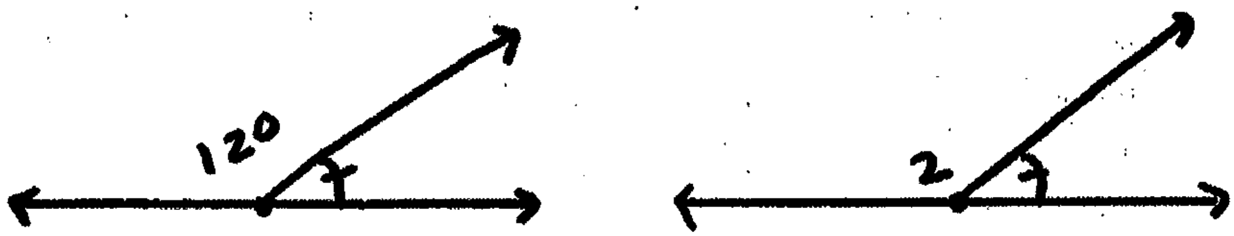
Example #2: Find Angle #1



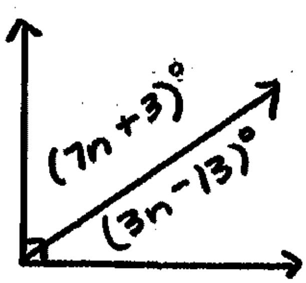
$180-56 = 124°$

Example #3: Find Angle #2

$\angle 2 = 120°$



Example #4: Find the value of n.



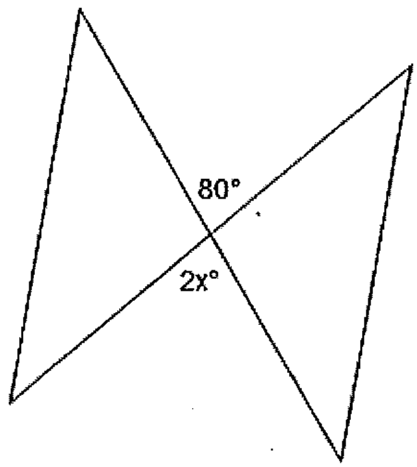
$7n+3+3n-13 = 90°$

$10n - \cancel{10} = 90$

$\quad +10 \quad +10$

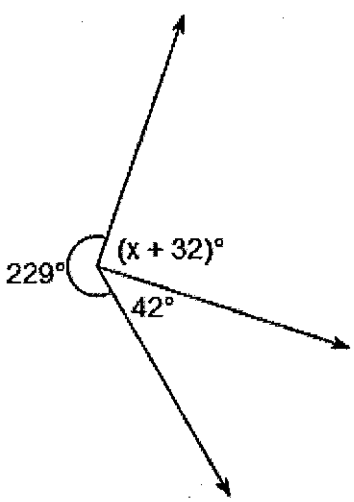
$\frac{10n}{10} = \frac{100}{10}$

$n = 1$



$$\frac{2x}{2} = \frac{80}{2}$$

$$x = 40$$



$$229 + x + 32 + 42 = 360$$

$$229 + x + 74 = 360$$

$$\begin{aligned} 303 + x &= 360 \\ -303 \quad &\quad -303 \end{aligned}$$

$$x = 57$$

HW: Worksheet